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APPRAISAL OF STOCHASTIC PRICING FORMULA FOR HYBRID EQUITY WARRANTS

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Abstract:

An evaluator of warrant models is essential for enhancing the current procedures used in the country's stock market and economy. This would provide additional insight for investors to make informed investment decisions and help to better understand the dynamics of warrant pricing in Malaysia. It has been established that the conventional process for valuing equity warrants using call option pricing models, like the Black-Scholes model, has numerous flaws, including the presumption of constant volatility and interest rates. As a matter of fact, existing alternative models were more concerned with providing techniques for pricing than with conducting empirical tests of warrant pricing models. Consequently, it is crucial to have a mathematical model that takes into account stochastic volatility and stochastic interest rates when pricing and analyzing equity warrants. Recent research by Roslan, Ibrahim, Jameel, and Ibrahim (2020) and Roslan, Ibrahim, Karim (2020) examined the pricing of hybrid equity warrants in the presence of stochastic factors. There was no comparative analysis conducted in these studies to ascertain the relative efficacy of the two methods used. This study's main goal is to empirically investigate how well equity warrant pricing models perform in the Malaysian market. This evaluation will take into account the impact of stochastic interest rates, using the Cox-Ingersoll-Ross (CIR) stochastic interest rate model, and stochastic volatility, using the Heston stochastic volatility model. To perform the analysis, the study will make use of a number of mathematical techniques, such as partial differential equations, Fourier transform approaches, and the change of measure method. The research methodology is divided into three main phases. It is anticipated that the research's findings will provide new perspectives that go beyond the domain of warrant pricing, creating chances for more widespread applications in fields like foreign exchange, insurance, and mortgage contracts.

Keywords:

Equity Warrants, Heston-CIR Model, Hybrid Models, Stochastic Volatility, Stochastic Interest Rate

Introduction

A product listed under the securities in Bursa Malaysia, the warrant market is quite significant in the financial industry in addition to the well-known stock market. Basically, there exist two types of warrants, known as equity warrants and structured warrants. Equity warrants are issued by the firm itself and grant holders the option, but not the duty, to subscribe for new common shares within a predetermined time frame at a predetermined price. Meanwhile a structured warrants are issued by third party issuers. According to Bursa Malaysia (2013), these include call warrants, put warrants, callable bull certificate warrants and callable bear certificate warrants. Purchasing warrants offers investors a number of benefits, including the ability to lock in the price of the underlying shares until the warrant's maturity date, lower transaction costs and commissions than share investment, and the opportunity to profit with losses strictly limited to the cost of investing. Stock warrants should be modelled as a mix of stochastic interest rate and stochastic volatility because the conventional method of valuing equity warrants using call option pricing models, such as the Black-Scholes model, has numerous drawbacks.

The prices derived from the Black-Scholes model, according to Xiao et al. (2012), tended to undervalue the equity warrants. This occurred because the exercise of an equity warrant may have an impact on the value of all other claims made against the company due to changes in the firm's capital structure and overall value. In addition, Lauterbach and Schultz (1990) highlighted that the BlackScholes model assumes that warrants are European, which holds the property of expiring at a given expiration date. When the underlying stock provides sufficiently high dividends, warrant holders—just like option holders—may actually decide to exercise their warrants early. As a matter of fact, there were a number of other related issues with applying the Black-Scholes model, including constant volatility, constant interest rates, and inversely correlated stock volatilities. Ukhov (2004) and Xiao et al. (2012) identified another drawback of applying the Black-Scholes model, which is the information required regarding the firm's worth and its variance, which are in fact unobservable. Furthermore, Schulz & Trautmann (1994) highlighted the significance of modelling equity warrants as a hybrid model of stochastic volatility and stochastic interest rate, suggesting that out-of-the-money warrants be modelled in a very general setting by incorporating both stochastic volatility and stochastic interest rates. In addition, equity warrants often have longer maturities than options.

The term of a warrant is typically stated in years rather than months. Because the short rate fluctuates arbitrarily over time, the typical practise of assuming a constant interest rate is obviously false. Indeed, it appeared that the underlying principles of the economy were the source of the widespread occurrence of long-term dependency in short-term interest rates

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(Lauterbach & Schultz, 1990; Yagi & Sawaki, 2010; Xiao et al., 2014). In financial models, the practical significance of including a stochastic interest rate was demonstrated by Roslan et al. (2014) and Cao et al (2016). But in the instance of warrant pricing, Xiao et al. (2014) conducted the sole current study on equity warrant pricing under stochastic interest rate fluctuations. Several authors also stressed the idea of adding stochastic volatility in connection with this issue.

According to Ukhov (2004), the Black-Scholes model could not be suitable for warrants since it makes the assumption that the default free bond's variance is constant. Moreover, the Black-Scholes model's constant variance assumption was cited by Lauterbach and Schultz (1990) as its most significant flaw. The aforementioned considerations highlight the pressing need to create a new pricing model for equity warrants that takes stochastic interest rates and volatility into account. By combining stochastic volatility and stochastic interest rate hybridizations, this model can both improve market characterization and lessen the errors caused by the use of constant interest rates. Thus, it is imperative to examine how the Cox-Ingersoll-Ross (CIR) model's stochastic interest rate and the Heston model's stochastic volatility are included into the pricing formulation for equity warrants in Malaysia. Although Roslan, Ibrahim, Jameel, and Ibrahim (2020) and Roslan, Ibrahim, and Karim (2020) conducted analyses for pricing hybrid equity warrants with these stochastic features, the relative merits of one over the other have not yet been addressed.

Literature Review

The Black-Scholes model represents a significant advancement in mathematical theory within the financial industry. This Nobel Prize-winning work, which was first presented by Black, Scholes, and Merton in 1973, offers a theoretical framework for pricing European options with geometric Brownian motion and constant volatility. At any point before to maturity, the value of the portfolio is determined using conditional expectations and change of probability metrics. Numerous scholars have expanded the Black-Scholes model to include various facets of the pricing and hedging of derivatives. Among these is the warrant valuation, which since 1980 has been raised as a significant portion of the corporate capital tied to bond issuance. In fact, according to Black and Scholes, this approach may be applied in various situations as an approximation to provide an estimate of the warrant value.

An equity warrant is the right to purchase the underlying share on a predetermined date and at a predetermined price. Although it is defined similarly to a call option, there are a few differences in its features that set it apart from a call option. A call option is issued by an individual, while a warrant is issued by a firm, according to Schulz and Trautmann (1994). By issuing new shares upon exercising a warrant, the issuing company's assets will eventually rise. The dividend and equity will be somewhat diluted as a result. Second, while warrants usually have maturities of at least many years, call options expire within a few months, and warrant volatility also affects the warrant's longevity. Furthermore, the firm's volatility and the underlying asset of a warrant, which is the warrant's value, are both required for warrant pricing and cannot be immediately viewed.

For the past forty years, scholars have been concentrating in the literature on creating appropriate techniques for assessing warrants. Analytical and numerical approaches are the two primary groups into which these techniques can be divided. Galai and Schneller (1978) were the first to propose an adjustment for the diluting effect of warrants on the firm's present value in terms of analytical methodologies. Following Black and Scholes' (1973) theory, which

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values a warrant as an option to purchase a share of the company's equity rather than a call option on ordinary stock, they developed an explicit formula for warrant pricing. Burney and Moore carried out research in 1997 to define and evaluate callable warrants.

They modelled callable warrants as warrants that might be called when stock prices reached multiples of exercise prices, based on the same idea of mitigating the dilution consequences. This leads to a semi-closed warrant pricing formulation and emphasises how important callable characteristics are to warrants. Warrants, however, might equally be viewed as options on the company's equity and do not require dilution correction. Dilution effects are what caused this, and the stock price ought to fully reflect them (Bajo & Barbi, 2010). These authors showed that the modelling of companies with outstanding warrants could be very well described by a constant elasticity of variance (CEV) process with certain elasticity values. Some authors also used numerical methodologies in their research, in addition to the previously described analytical approaches. Several optimisation problems have been resolved by the application of diverse numerical techniques.

Using the finite-difference method, Schulz and Trautmann (1994) looked at the problem of non-dilution adjusted valuation for a company that had outstanding warrants. They verified that, when using the stock price as the state variable in the American constant variance diffusion model, there is essentially no dilution-related pricing bias. One disparity that was discovered, though, was that unseen data was being employed in the equations to determine the firm's value. Ukhov (2004) came up with a workable approach that year that involved solving an iterative algorithm numerically using observable variables. With the volatility of the stock price and return, this was possible. However, in order to examine the impact of a possible wealth transfer from equity holders to debt holders, a study on the relationship between a firm's capital structure and future investment decisions was carried out (Crouchy and Galai, 1994). They asserted that the risk inherent in equity was influenced by the functions of debt, warrant conditions, and maturity period.

Recently, a number of scholars have presented complete genetic algorithms, hybrid intelligent algorithms, and central time and space schemes in their warrant pricing systems, respectively. These authors are Xiao et al. (2012), Xiao et al. (2014), and Mansor and Jaffar (2014). Yagi and Sawaki (2010) used the binomial method in a game option strategy to determine the callable warrant value. Meanwhile, Ismail, M. N. (2021) also determined the call warrant's price using a binomial model. When the volatility is stochastic, Marin, Pareja, and Manzur (2021) value real warrants using the multiplicative quadrinomial tree numerical technique with non-constant volatility, which is based on stochastic differential equations of the GARCH-diffusion type. In the G.A.R.C.H. diffusion model, Wu, Zhou, and Wang (2018) employed a quasi-closed form pricing formula for European warrants, which significantly eases the calculating warrant prices. To the best of our knowledge, despite the presentation of many stock warrant pricing models above, the appraisal process for equity warrant pricing that takes into account hybrid models of stochastic volatility and interest rates has not yet been completed.

There has only been one study recently conducted in China on the pricing of equity warrants under stochastic interest rates (Xiao et al., 2014). By using the change of measure technique and the Fourier transform of generalised function approaches, Roslan, Ibrahim, and Karim (2020) provided the characteristic function for warrant pricing under the Heston and CIR models. According to Ma, Yue, Wu, and Ma (2020), the fast Fourier transform (FFT) algorithm offers a quick, accurate, and simple way to determine warrant prices.

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In a different study, Roslan, Jameel, and Ibrahim (2020) used partial differential equation (PDE) techniques to establish the warrant pricing formula. The assessment of stock warrant prices in Malaysia through the comparison of Roslan, Ibrahim, and Karim (2020) and Roslan, Jameel, and Ibrahim (2020) studies is what makes this research novel. It does this by identifying the optimal stochastic pricing formulas. By hybridising stochastic volatility and stochastic interest rate, this research not only improves market characterization but also lessens the errors caused by using a fixed interest rate when pricing stock warrants. This will be very helpful to Malaysian investors in equity warrants who are looking for a commonly used model that local investment experts may use to evaluate equity warrants.

Research Methodology

This study's primary goal is to evaluate Malaysian hybrid equity warrant pricing using two stochastic models of Roslan, Ibrahim and Karim (2020) and Roslan, Jameel and Ibrahim (2020). There are three stages to this research's technique. The first stage is called setup which consists of gathering data for preliminary modelling as well as reviewing relevant literature. In the second phase, computer tests and calibration will be conducted to assess the model. Ultimately, the outcomes will be examined in the final stage. The following is a description of the detailed procedures:

Stage 1: Setup

Data for this study will be gathered in the first phase from two sources: a selection of open data and Bursa Malaysia. Due to their lengthier lengths until expiration, only warrants in the European and American styles will be chosen. The emphasis will be on equity warrants that are actively traded.

Stage 2: Computer Tests and Calibration

Here, a pilot run and the development of computer codes will be the initial steps in examining the involved models' performance. Following their completion, the validation procedure will compare the models' prices with the actual warrant prices from Bursa Malaysia. Enhancing the models' usability and functionality is the goal.

Stage 3: Analysis of Findings

At this point, additional examination will be done to evaluate how well the involved models perform in terms of price errors and execution time. There will be comparisons with various models of warrant pricing in the literature, e.g., the Lauterbach-Schultz, Ukhov, and Black-Scholes models. The mean bias (MBIAS), mean absolute error (MAE), mean absolute percentage error (MAPE), and root mean square error (RMSE) are a few metrics that can be used to assess how well these models are performing. In terms of execution time, the optimal pricing model will be the one with the shortest execution time ever recorded.

Conclusion

Using the change of measure technique and Fourier transform approaches, this study intends to investigate empirical performances of the pricing formula for equities warrants in Malaysia under the Heston stochastic volatility and CIR stochastic interest models. The results of this study will be very helpful to Malaysian equity warrant investors who are looking for a model that is commonly utilised by local investment analysts to evaluate equity warrants.

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