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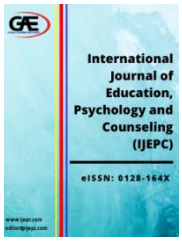


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# DEVELOPMENT OF MATHEMATICS READING ASSESSMENT: PSYCHOMETRIC EVALUATION BASED ON SEM AND IRT

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**Abstract:**

The purpose of this study is to evaluate the psychometric properties of mathematics reading assessment based on structural equation model (SEM) and item response theory (IRT). The mathematics reading assessment is conducted in the form of text-reading related to real-life probability and statistics issues. The sample is 789 sixth graders from 13 primary schools in Taiwan. The latent construct of mathematics reading includes three factors which are based on the content area reading components provided by M. C. McKenna and R. D. Robinson. These three factors of mathematics reading assessment in this study are general reading comprehension, prior knowledge of mathematics, and mathematics-specific skills respectively. Firstly, the researcher develops the mathematics reading assessment and confirms the item difficulties, item discriminations, and reliability of assessment. A structural equation model (SEM) is adopted to test the factor construct of the mathematics reading assessment. According to the analysis, it shows that the item difficulties, item discriminations, and reliability are very well. The structural equation model reveals there exist three factors that conform to the assumption of content area reading components mentioned above. Secondly, this study calibrates item characteristics by the three-parameter logistic model of item response theory (IRT). Results indicate that all item fitness performs well. Items belonging to prior knowledge of mathematics have the highest item discriminations. Items of mathematics-specific skills have the highest item difficulties. This study establishes a well-structured mathematics reading assessment. Results could also provide references for instruction and assessment of mathematics reading. Finally, based on the findings of this study, some recommendations and suggestions for future researches and methodologies are also discussed.

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**Keywords:**

Item Response Theory, Mathematics Literacy, Mathematics Reading, Statistics Literacy, Structural Equation Model

## Introduction

This study aims to validate mathematics reading assessment based on confirmatory factor analysis and three-parameter logistic model of item response theory. Mathematics is the basic subject of all disciplines. Mathematics is the foundation of many disciplines because it provides logical thinking and algorithmic basis to solve problems. Text reading is the important ability for human learning because reading activities develop the mind to understand the symbolic words in order to realize meanings. International assessment like PISA(Program for International Student Assessment) and TIMSS(Trends in. International Mathematics and Science Study) also indicate mathematics is the common core the many subjects. Although TIMSS and PISA assess different things, TIMSS is more curriculum-based to school education whilst PISA is the literacy-based to real life problems. Both of them suggest that mathematics reading is an important issue because it is deeply related to mathematics achievement and literacy (Rindermann, 2007). PIRLS 2011 (2011 Progress in International Reading Literacy Study) also suggests that students who have not learned the basic skills of mathematics and reading by the end of their fourth year of schooling are at risk for academic failure in the future (Martin & Mullis, 2013). Therefore, mathematics as well as reading skills are essential for successful functioning in all aspects of 21st century life.

In addition to the reading and comprehension about word problems in mathematics textbooks, one branch of mathematics reading is to investigate the mathematics reading about real-life texts. Since mathematics and reading skills are necessary beyond the school, in real-life situations. Many evidences indicate that there are individual differences and difficulties in mathematics reading for students. There has been also growing interest to investigate the components structure of mathematics reading (Harlaar, Kovas, Dale, Petrill, and Plomin, 2012). However, little is known about the factor structure of components related to mathematics reading (Adams, 2003).

In accordance with the viewpoints mentioned above, it is feasible to develop mathematics reading assessment and investigate its factor structure as well as item characteristics. This study aims to develop mathematics reading assessment for primary school students based on three components of mathematics reading, which is provided by M. C. McKenna and R. D. Robinson to explain content-related mathematics texts reading comprehension (Mckenna and Robinson, 2002). This study wants to validate the confirmatory factory structure of mathematics reading assessment based on structural equation model (SEM). Moreover, this study also calibrates its item parameters by using three-parameter logistic model of item response theory. This study is an integrated methodology to investigate the characteristics of mathematics reading assessment. The results could provide references for instruction and assessment as well as future studies related to mathematics reading (Purpura, Napoli, Wehrspann, & Gold, 2017).

## Literature Review

Literatures about theoretical foundations of mathematics and reading will be discussed. In accordance with literacy related to mathematics and reading, the literatures will discuss the importance of mathematics reading and its assessment.

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***Theoretical Foundation of Mathematics and Reading***

Nowadays mathematics is considered the foundation of many disciplines, such as natural science and engineering, because mathematics improves logical thinking and algorithmic so as to solve problems. Reading is the important activities of learning because it develops the mindset to understand the written contexts. Reading comprehension plays an important role in the process of learning activities. Reading activities also have influential contributions to acquisition and communication of mathematics knowledge. Hence, reading has long been an important issue in educational research (Vacca and Vacca, 2002). Moreover, Fenwick (2001) emphasizes instruction of reading strategies and comprehension in classroom is essential. As teachers, they must know how to adopt proper cognition guidance so that students can improve reading and comprehension. That is, instruction for specific domain reading skills, such as mathematics reading, should be emphasized in classroom instruction (Bossé & Faulconer, 2008).

Reading in content area is also connected to literacy performance. Mathematics reading means the reading of mathematical context, symbol, vocabulary and graph. Reading mathematics is a multifaceted task because the reader is challenged to acquire understanding and comprehension of mathematical meanings (Adams, 2003). Readers must have mathematical understanding with proficiency through the reading of numerals and symbols as well as vocabulary. Brozo, Shield, and Topping (2007) shows that students' weakness in their mathematics reading is often due to the obstacles they face in these complex symbols as they attempt to comprehend read the symbols. Therefore, students in compulsory education need more guidance to adopt flexible reading skills to learn how to read mathematics. Once students have difficulties in mathematics reading, they begin to have learning degradation in mathematics. Mathematics reading is an important skill and activities to gain mathematics concepts and knowledge.

***Mathematics Reading Assessment and Literacy***

In terms of mathematics and reading assessment as well as literacy, PISA, TIMSS and PIRLS pay much attention to the fields of mathematics and reading. The reason is because both mathematics and reading are important literacy of national citizens. The report of PISA constructs five levels to describe student performance in reading literacy. PISA indicates that "A country with the number of fifth level reading ability of citizens is an important indicator of the country's future competitiveness." In accordance with this viewpoint, many countries give great attention students' reading activities and performance and they also emphasize the importance of subject reading. Therefore, one consensus is that the fundamental premise for "learning to read" in order to "reading to learn". Harlaar, Dale and Plomin (2007) indicates that "learning to read" and "reading to learn" could occur simultaneously and continually throughout years of education. Students need "learning to read" and "reading to learning" instruction, especially within content areas such as mathematics, so that they can be more solid in mathematics knowledge.

Purpura and Reid (2016) mention that quite a few literatures show that individual differences in reading and mathematics skills. However, it is still unclear whether mathematics is differentially related to word decoding and reading comprehension. Cognitive components of content reading influence the reading comprehension. Barton, Heidema, and Jordan (2002) figure out mathematics reading happens not only in mathematics textbooks in which mathematics problems are needed to be solved, but also in many kinds of texts which contains mathematics elements. In terms of mathematics reading, mathematical vocabulary and words

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have its unique way in sentences and paragraphs in which are difficult for students to understand, One common conclusion is that psychological and cognitive mechanisms in mathematics reading should be further explored. Mckenna and Robinson (2002) show that writers and readers are difference roles to transform meanings from writing to reading. They propose the definition of content-area reading. As to content-area reading, readers must have three cognitive components of content literacy. As shown in Figure 1, the three cognitive components are general literacy skills, prior knowledge of content and content-specific literacy skills. The intersection of these three cognitive components is called content literacy. Based on the viewpoint of M. C. Mckenna and R. D. Robinson, this study proposes mathematics reading in terms of three factors. They are general reading comprehension, prior knowledge of mathematics, and specific skills of mathematics. Borasi and Siegel (2000) propose it is essential to develop text-reading in daily life so as to evaluate students' performance. Hence, this study adopts the above definition to develop the mathematics reading text and assessment.

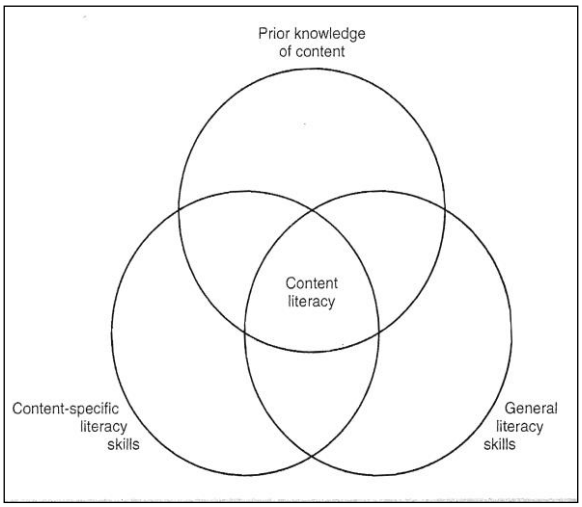


**Figure 1: Cognitive Components of Content Literacy**
Source: " Teaching through text-reading and writing in the content area", Mckenna & Robinson, 2002, p.9

## Research Design and Methodology

This study integrates the theoretical foundation of reading in content area and mathematics reading. Three factors which are general reading comprehension, prior knowledge of mathematics, and specific skills of mathematics establishes the subtests of research instrument. The research develops the mathematics reading assessment and calibrates the psychometrical properties by using structural equation model, which is confirmatory factor analysis, and item response theory, which is three-parameter logistics model, respectively.

### *Research Design*

According to the purpose of this study, research design and procedure of this study is shown as Figure 2. As depicted in Figure 2, the mathematics reading assessment gives the feasible instrument to evaluate students' performance. The two analytic method, which are structural equation model and item response theory, will calibrate the properties of the mathematics reading assessment.

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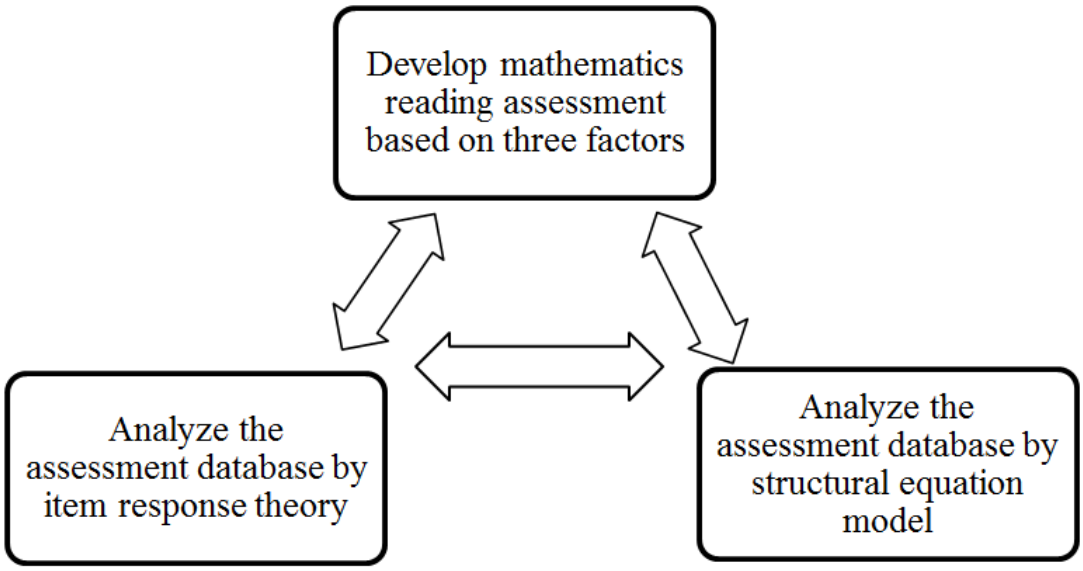


**Figure 2: Research Design and Procedure**

***Sample and Assessment Data***

This study develops mathematics reading assessment related to the content of statistics and probability in daily life. In each factor of mathematics reading, there is one short article and 5 items to evaluate students' performance. Namely, there are totally three short articles and 15 items. All these items are multiple-choice items. The Cronbach reliability is 0.87 and its internal consistency is acceptable. The sample 789 sixth graders from Taiwan and they are from 13 elementary schools. Students must read each article firstly and then respond the corresponding items.

**Results and Discussions**

There are two major parts of discussions according to the purpose of this study. Firstly, confirmatory factor analysis of structural equation model is discussed to show the construct validity of mathematics reading assessment. Secondly, three-parameter logistic model of item response theory is to calibrate the item characteristics and test information.

***Confirmatory Factor Analysis of Structural Equation model***

The model fit is evaluated with a Chi-square test as well as other CFA fit indices, including Root Mean Square Error of Approximation (RMSEA), Comparative fit index (CFI), and Tucker Lewis index (TLI). The model fit is conducted to test the fit of the data obtained from the mathematics reading assessment to the hypothesized factor structure. As shown in Figure 3, factor loadings were reasonably high. As expected from the fact, the Chi-square test is known for its sensitivity to a large sample size. Therefore, the result of the Chi-square test is 323.877 ($df=87$, $p<0.001$) and it is significant. The other fit indices reveal that the confirmatory factor analysis is a good model fit to the sample data. They are RMSEA= 0.059, CFI= 0.977, and TLI= 0.972. Based on these indices, it means that hypothesized factor structure is supported and that the mathematics reading assessment measures three factors mentioned above.

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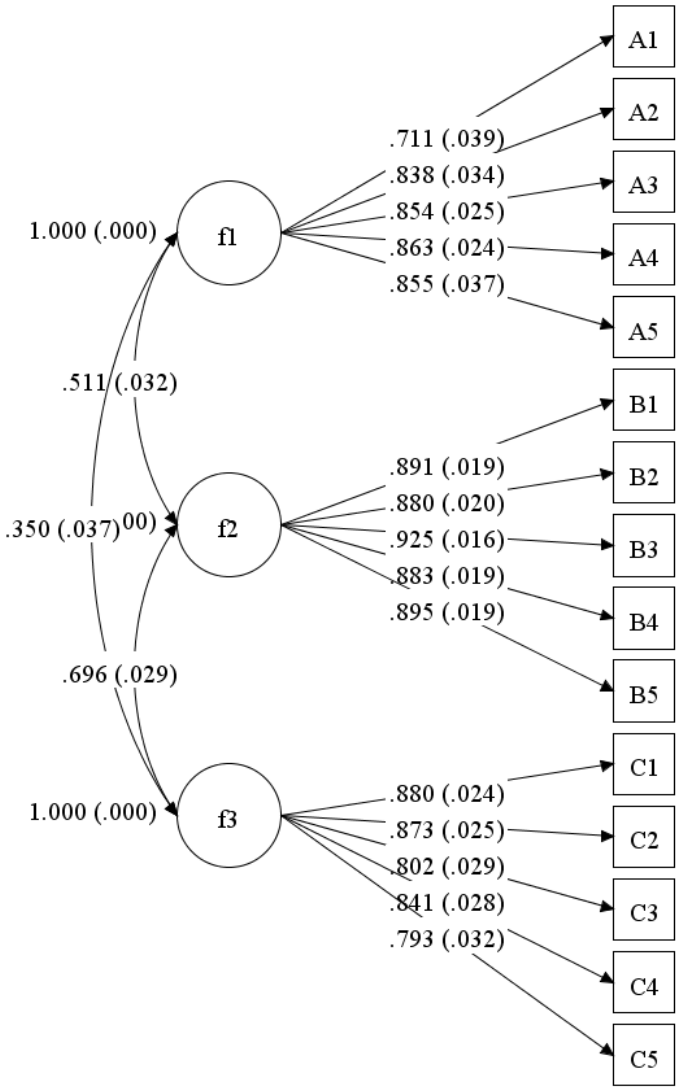


**Figure 3: Confirmatory Factor Analysis of Mathematics Reading Assessment**

Table 1 depicts the mean and standard deviation of each factor score. It shows that students have the highest performance in general reading comprehension and the lowest performance in mathematic-specific skills. This finding conform most research because general reading comprehension is the common core in reading activities but mathematics-specific skills need more mathematics knowledge and concepts.

**Table 1: Descriptive Statistics of Factor Scores**

| Factors Scores | Mean | SD |
|---|---|---|
| f1 (general reading comprehension) | 3.562 | 1.622 |
| f2 (prior knowledge of mathematics) | 2.240 | 2.022 |
| f3 (mathematics-specific skills) | 1.340 | 1.661 |
| Total | 7.141 | 4.088 |

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Table 2 shows the correlation coefficients among factor scores. One can find that there exist positive correlations among all factors. Furtherly, we can also indicate these each factor owns its specific cognitive components but these components are positively correlated.

**Table 2: Correlation Coefficients among Factor Scores**

| | | Factors | | | |
|---|---|---|---|---|---|
| | | f1 | f2 | f3 | total |
| **Factors** | f1 | - | .370** | .234** | .675** |
| | f2 | | - | .527** | .856** |
| | f3 | | | - | .760** |
| | total | | | | - |

**$p$<.01

***Item Calibration and Test Information based on Item Response Theory***

The item format of mathematics reading assessment is multiple choice items. Hence, three-parameter logistic model is suitable to analyse item characteristics. The formula of three-parameter logistic model is as follows (Baker & Kim, 2004). The three-parameter logistic model means that the response probability of item $i$ is $P_i(\theta)$ in which the latent trait (ability) of task-taker is . The item parameters are discrimination parameter $a_i$ , difficulty parameter $b_i$ , and guessing parameter $c_i$ .

$$P_i(\theta) = c_i + (1 - c_i)\frac{1}{1 + e^{-a_i(\theta - b_i)}}$$

In terms of the distribution of latent trait (ability), Figure 4 display the box-whisker graph. Moreover, Kolmogorov-Smirnov test is 0.128 (p<.01). It means that the distribution of latent trait (ability) is not normal distribution and it is negative skewness as well as negative kurtosis.

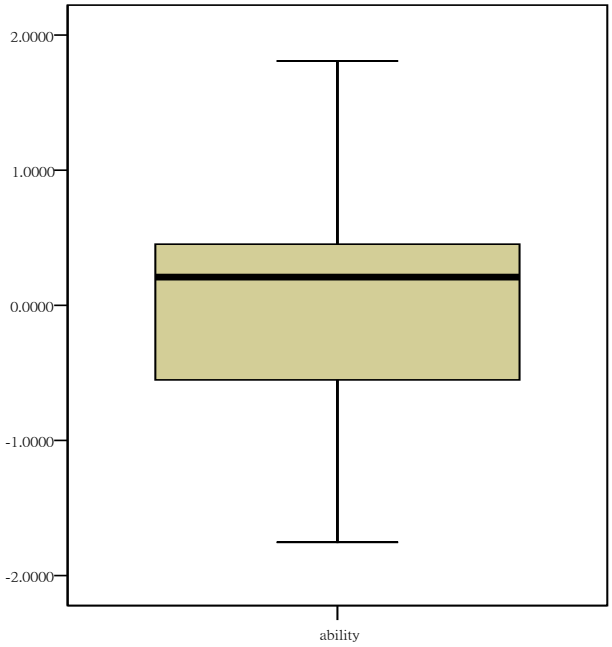


**Figure 4: The Box-Whisker Graph of Ability**

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In Table 3, it shows items of f2(prior knowledge of mathematics) have quite high item discrimination in contrast to items of f1 (general reading comprehension). As to item difficulty, items of f3 (mathematics-specific skills) have high item difficulty in contrast to items of f1 (general reading comprehension). As for item guessing, all the items have low guessing parameters and they are almost the same.

**Table 3: Item Parameters of Mathematics Reading Assessment**

| Factors | Item | a-parameter | b-parameter | c-parameter |
|---|---|---|---|---|
| f1 (general reading comprehension) | A1 | 0.74 | -0.82 | 0.10 |
| | A2 | 0.88 | -0.96 | 0.09 |
| | A3 | 0.76 | -0.33 | 0.08 |
| | A4 | 0.78 | -0.53 | 0.08 |
| | A5 | 0.93 | -1.13 | 0.09 |
| f2 (prior knowledge of mathematics) | B1 | 2.15 | 0.39 | 0.06 |
| | B2 | 2.21 | 0.25 | 0.09 |
| | B3 | 2.52 | 0.31 | 0.06 |
| | B4 | 2.34 | 0.33 | 0.11 |
| | B5 | 2.10 | 0.35 | 0.04 |
| f3 (mathematics-specific skills) | C1 | 2.17 | 0.80 | 0.10 |
| | C2 | 1.81 | 0.78 | 0.08 |
| | C3 | 1.50 | 0.87 | 0.08 |
| | C4 | 1.95 | 1.03 | 0.06 |
| | C5 | 1.32 | 1.11 | 0.04 |

Item response theory brings contributions of item information which is the extension of the concept of reliability in accordance with Fisher information theory. Information is a function of the model parameters (de Ayala, 2009). Test information is the summation of all item information. Test information represent the precision in which the test .provides for all levels of latent traits (abilities). Let $Q_i(\theta) = 1 - P_i(\theta)$and the item information of item $i$ , $I_i(\theta)$ is as follows. From Figure 5 to Figure 7, one is concluded that subtest of general reading comprehension could give more test information for students of lower level ability and subtest of prior knowledge of mathematics could give more test information for students of middle level ability. Besides, subtest of mathematics-specific skills provide the most test information for students of high level ability.

$$I_i(\theta) = a_i^2 \frac{Q_i(\theta)}{P_i(\theta)} \left[ \frac{P_i(\theta) - c_i}{1 - c_i} \right]^2$$

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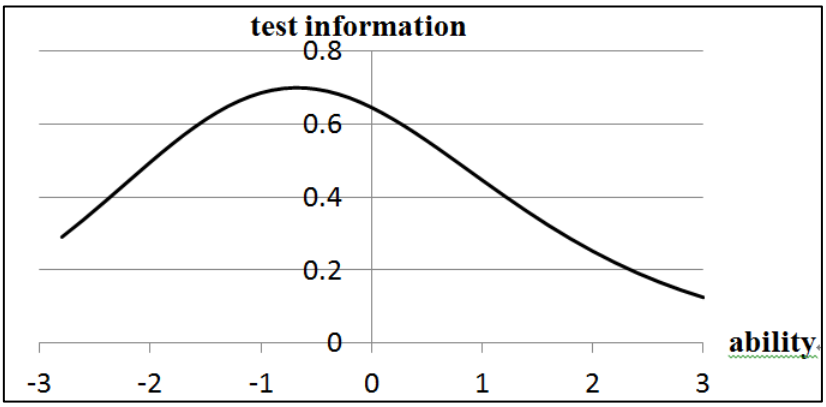


**Figure 5: Test Information Curve for the Factor of General Reading Comprehension**

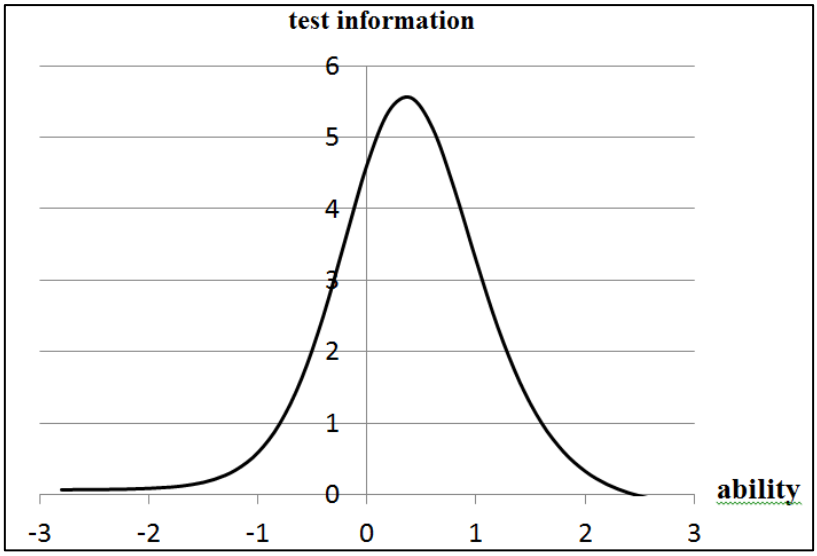


**Figure 6: Test Information Curve for the Factor of Prior Knowledge of Mathematics**

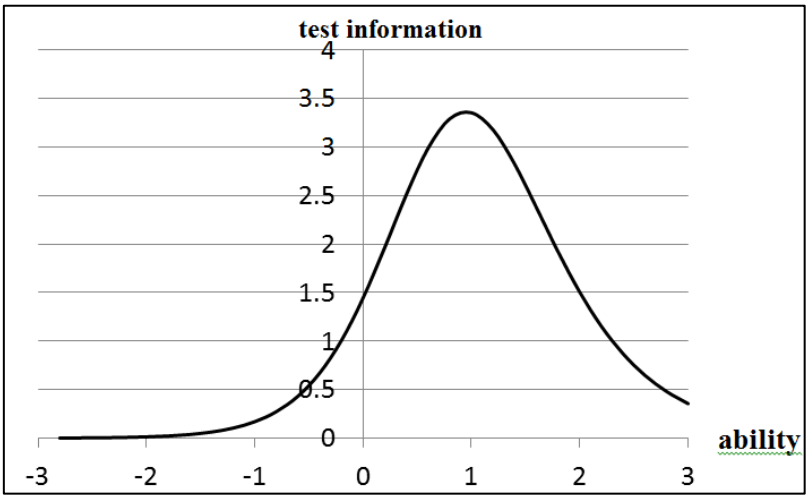


**Figure 7: Test Information Curve for the Factor of Mathematics-Specific Skills**

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## Conclusions

The purpose of this study is to explore the validity of mathematics reading assessment and analyse item characteristics. Findings of confirmatory factor analysis shows the assessment own well-structured construct validity. On the other hand, the analyses of three-parameter logistic model tell us that item characteristics and test information vary with dimensions of mathematic reading. Findings of this study support the theory foundation of content-area reading related to mathematics (Mckenna & Robinson, 2002). That is, the mathematics reading consists of three factors which are general reading comprehension, prior knowledge of mathematics, and specific skills of mathematics. In addition, this study shows prior knowledge of mathematics is the basis of mathematics reading and mathematics-specific skills affect students' performance greatly. Results of this study coincide with recent literature (Schöber, Schütte, Köller, McElvany, & Gebauer, 2018).

In sum, the analyses support the feasibility of mathematics reading assessment for elementary school students and provide the quantitative methodologies. Except for topic of statistics and probability, this study could provide recommendations for instruction and assessment of mathematics reading. Future studies could develop mathematics reading assessment for secondary school students based on other topic of mathematics.

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