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NUMERICAL METHODS IN CRACK MATERIAL DETECTION AND ANALYSIS: A SYSTEMATIC REVIEW

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Abstract:

This systematic literature review examines recent advancements in numerical methods for crack detection and analysis, highlighting their increasing accuracy and computational efficiency. Using the PRISMA framework, an extensive search conducted in Scopus and Web of Science databases (2023–2024) identified 15 primary studies focusing on three main themes: numerical methods for crack detection, dynamic crack propagation analysis, and advanced material-specific fracture analysis techniques. Key findings indicate substantial improvements, particularly in hybrid numerical approaches like Extended Finite Element Method (XFEM), Physics-Informed Neural Networks (PINNs), and phase-field methods, which significantly enhance real-time crack detection capabilities in critical structural applications, including rotating machinery, fluid pipelines, and composite materials. Despite these improvements, several limitations persist, notably the difficulty in generalizing results across different materials, computational challenges when scaling models for large-scale industrial applications, and dependence on material-specific parameters requiring further experimental validation. Future research should prioritize developing universally adaptable numerical models that effectively integrate these advanced techniques to address these limitations, facilitating broader applicability and reliability in structural health monitoring.

Keywords:

Numerical Methods, Crack Detection, Structural Health Monitoring, Phase-Field Models, Physics-Informed Neural Networks (PINNs)

Introduction

Crack detection and analysis are critical for ensuring the structural integrity and safety of various engineering structures. Over the years, numerous numerical methods have been developed to identify and analyze cracks in materials and structures (Scamardo, Franchi, and Crespi 2022; Zhao and Zhang 2020). These methods range from traditional finite element methods (FEM) to more advanced techniques such as the eXtended Finite Element Method (XFEM), numerical manifold methods (NMM), and artificial intelligence-based approaches (Chai, Lv, and Bao 2020; Chen, Lai, and Lin 2020; Gu et al. 2023; Liu et al. 2021). Each method offers unique advantages in terms of accuracy, computational efficiency, and applicability to different types of crack problems.

One of the prominent methods in crack analysis is the Numerical Manifold Method (NMM), which has shown significant success in modeling complex discontinuities. NMM employs both physical and mathematical meshes to simplify the meshing task and directly capture discontinuities across crack surfaces without additional unknowns at the nodes (Lv et al. 2020; Wang et al. 2019). This method has been further enhanced by incorporating strain smoothing techniques to improve accuracy and efficiency, particularly in dynamic crack analyses. Additionally, the use of explicit time integration and mass lumping strategies in NMM has proven beneficial for achieving stable and accurate results in both static and dynamic scenarios (Mate-Kole, Margot, and Dewji 2023; Orie and Ogbonna 2024; Shoheib et al. 2022).

Artificial intelligence and optimization algorithms have also been integrated with numerical methods to enhance crack detection capabilities. For instance, the combination of Artificial Neural Networks (Tikhomirov 2020) with the Jaya algorithm has been used to improve the accuracy of crack identification in plates by leveraging dynamic and static datasets from eXtended IsoGeometric Analysis (XIGA) (Markov and Kanaun 2018). Similarly, hybrid approaches that combine deep learning models with Bayesian probabilistic analysis have been developed for robust crack detection in noisy environments (Su, Cai, and Xu 2023). These advanced techniques not only improve the precision of crack detection but also offer the potential for real-time monitoring and assessment of structural health.

Despite substantial advancements in numerical methods such as the Numerical Manifold Method (NMM), Extended Finite Element Method (XFEM), and Physics-Informed Neural Networks (PINNs), existing methods still encounter significant challenges. Particularly, there is a notable limitation in achieving optimal adaptability across varying structural materials and environmental conditions. Current numerical methods often require specific adaptations and extensive computational resources, restricting their scalability and universal applicability in real-world scenarios. Thus, there is a clear research gap concerning universally adaptable numerical approaches that maintain high computational efficiency and accuracy across diverse structural and material conditions.

This systematic literature review aims to (1) identify the recent advancements and limitations of existing numerical methods in crack detection and analysis, (2) critically evaluate the effectiveness of these methods in addressing dynamic loading conditions and crack propagation in complex, high-stress environments, and (3) explore advanced numerical fracture analysis techniques tailored to specific materials, emphasizing their applicability in real-world structural engineering scenarios.

In summary, the field of crack detection and analysis has evolved significantly with the development of various numerical methods and their integration with artificial intelligence. Techniques such as NMM, XFEM, and hybrid AI-based approaches have demonstrated their effectiveness in accurately identifying and analyzing cracks in different structural contexts (Li et al. 2024). As research continues to advance, these methods are expected to become even more sophisticated, providing engineers with powerful tools for ensuring the safety and reliability of critical structures.

Literature Review

The study of numerical methods in crack detection and analysis has progressed across various materials and structural conditions, with advancements primarily focusing on the efficiency and adaptability of these methods for specific applications. After (Wang et al. 2024) present a modified domain-independent interaction energy integral method (IEIM) for analyzing thermal shock-induced cracks in nonhomogeneous materials, particularly those with inclusions. The method combines IEIM with the extended finite element method (XFEM) and finite difference method (FDM) to address complex thermal-shock crack issues in particulate composites. This approach demonstrates the importance of mixed-mode transient thermal stress intensity factors (TSIFs) in assessing the crack behavior in materials with inclusions. Similarly, (Li and Wang 2024) develop a numerical model for predicting fatigue life in liquid-storage tanks, with a focus on crack length and depth based on the Paris law. Their model validates theoretical and experimental data, underscoring the role of stress distribution analysis in structural durability assessments. (Qu, Gu, and Fan 2024) introduce a seven-phase mesoscale numerical method to examine chloride penetration in recycled aggregate concrete (RAC) with cracks, illustrating that crack characteristics, particularly length, significantly impact chloride penetration in cracked RAC. Together, these studies showcase the versatility of numerical methods in analyzing crack behavior across varying stress conditions and material types.

In the realm of complex crack singularities and finite element modeling, (Ju, Yu, and Zhou 2024) contribute a general algorithm for integrating three-dimensional crack singularities using partition-of-unity (PU)-based methods. Their approach, founded on the Duffy transformation, addresses the challenge of computational efficiency in PU-based methods, especially in 3D crack problems. The algorithm's efficiency is highlighted by its reduction in Gauss points required for precision, a critical development for high-resolution crack-front analysis. (Kushwaha and Patel 2023) extend these findings by introducing a non-standard finite element (FE) method that integrates mathematical programming to simulate tensile crack initiation in quasi-brittle materials without the need for remeshing. This method uses localized plastic deformation along FE edges, optimizing structural analysis by avoiding interface introduction typically required in standard FE approaches. Both studies emphasize innovative approaches for handling singularity and discontinuity, a recurring trend in numerical crack analysis aimed at improving model accuracy and computational efficiency.

The need for multi-scale, adaptable modeling techniques in composite material analysis is underscored in the work of (Ju et al. 2024; Kushwaha and Patel 2023; Wang et al. 2024). Zhang et al. demonstrate the effectiveness of combining IEIM, XFEM, and FDM in managing thermal-shock crack propagation in composite materials, while Lv et al. propose a 3D conformal preconditioning strategy to address edge and vertex singularities in cracked solids. Scamardo et al.'s method further complements this line of research by offering a simplified approach to crack propagation in quasi-brittle materials, highlighting the growing emphasis on

creating multi-functional numerical tools that can handle both micro- and macro-scale variations in composite structures. These studies collectively underscore the limitations of traditional methods and the necessity for novel, adaptable approaches in composite crack analysis.

Despite these advancements, significant knowledge gaps persist, particularly concerning the scalability and universality of numerical methods across diverse material compositions and stress conditions. (Wang et al. 2024) and (Li and Wang 2024) provide insights into crack behavior in nonhomogeneous materials and liquid-storage tanks, respectively, but their methods may require adaptation to account for more varied environmental conditions. The PU-based approach introduced by (Ju et al. 2024) enhances precision in three-dimensional crack singularity analysis, yet it remains computationally intensive, especially for large-scale industrial applications. (Kushwaha and Patel 2023) address computational efficiency through a non-standard FE method; however, the applicability of this method to more complex materials with mixed-mode cracks remains untested. These limitations indicate the need for future research to develop universally adaptable, efficient numerical tools capable of adjusting to the specific requirements of diverse material types and structural configurations.

The field of numerical methods in crack detection and analysis encompasses a diverse range of approaches and applications, with recent advancements focusing on the accuracy, efficiency, and adaptability of these methods for various structural challenges. (Guo, Ren, and He 2024; Kalay et al. 2024) propose a modified extended finite element method (XFEM) tailored to predict the nucleation and propagation of multiple cracks in plane members, with a particular emphasis on the energy dissipation during crack formation. This method introduces a crack competition criterion based on the minimum total energy principle, ensuring accurate sequencing of new-crack nucleation and existing-crack propagation. In comparison, (Kalay et al. 2024) develop a Bézier-based extended isogeometric analysis (XIGA) for thermoelastic-plastic crack propagation under welding residual stress and thermal loads, offering an accurate solution without requiring remeshing. Additionally, (Azinpour et al. 2023; W. Ji et al. 2024) demonstrate the benefits of using an adaptive integration step in high-speed calculations for steam cracking, significantly reducing computational time while maintaining accuracy. These studies collectively indicate a trend towards enhancing computational efficiency and robustness in multi-crack and multi-material scenarios.

Further advances in numerical methods highlight the importance of specialized models in fatigue and crack life prediction. (Azinpour et al. 2023) present a method for estimating the fatigue crack initiation size using elastic-plastic fracture mechanics (EPFM), focusing on the cumulative damage in short crack fatigue life under low-strain, cyclic torsional stress. This approach aligns with the studies of (Mutra, Mallikarjuna Reddy, and Babu Rao 2025), who investigate various methods for calculating the stress intensity factor (ΔK) in fretting cracks. By comparing indirect finite element (FE) simulations with coupled approaches, de Pannemaecker et al. conclude that the decoupled approach is faster for long cracks, while coupled approaches offer higher accuracy for shorter cracks. Similarly, (Li and Wang 2024) address fatigue life prediction in liquid-storage tanks by developing a numerical model that analyzes crack size based on stress distribution and hoop stress impact, verifying the model's feasibility through experimental data. These studies underscore the essential role of tailored numerical methods in accurately predicting fatigue life, especially in complex crack initiation and propagation scenarios.

In addition to addressing computational efficiency, recent research has emphasized the handling of complex crack geometries and material interfaces. (Kalay et al. 2024) introduce a Bézier-based XIGA method that effectively manages thermoelastic-plastic crack propagation, particularly useful in analyzing the impact of welding residual stress without the need for re-meshing. (W. Ji et al. 2024) further highlight the benefits of adaptive numerical integration in steam cracking scenarios, where varying reaction rates necessitate changes in integration steps for accurate, high-speed computations. (Hu et al. 2024) also contribute to this field by focusing on multiple crack interactions, using XFEM to determine crack growth based on energy dissipation per unit length. Together, these studies illustrate the importance of adaptable numerical models capable of addressing complex geometries and multi-material interfaces, a critical aspect of modern structural analysis and engineering.

Despite these advancements, several gaps remain in the current research, particularly in developing universally applicable models for multi-crack interactions across diverse materials. The adaptive integration method proposed by (W. Ji et al. 2024) offers significant computational benefits in specific scenarios like steam cracking but may require further validation across broader applications. (Azinpour et al. 2023) and de (Azinpour et al. 2023) highlight limitations in fatigue crack prediction models, suggesting a need for adaptable methods that can accommodate both long and short crack formations with precision. (Kalay et al. 2024) Bézier-based XIGA approach, while effective in specific thermoelastic contexts, may not fully address the complexities introduced by more intricate material heterogeneities or high-dimensional crack formations. These limitations point to a broader need for future research focused on developing adaptable, multi-functional numerical tools that seamlessly transition between varied crack scales, material compositions, and stress conditions.

The study of numerical methods for crack detection and analysis has made significant advancements in accurately predicting crack behavior in various materials under different conditions. (Guo et al. 2024) focus on the development of a numerical model to predict the initial crack load in hollow-cored concrete beams under mid-span loading. By integrating the modulus of rupture into the moment-curvature relationship, their approach combines experimental and numerical data, resulting in a 90.55% agreement between model predictions and experimental outcomes. Similarly, (Faridi, Roy, and Singhal 2024) addresses the calculation of stress intensity factors in both homogeneous and heterogeneous materials, offering algorithms based on asymptotic solutions for materials with crack interfaces between different media. This approach underscores the importance of accurately determining stress intensity to evaluate material fracture behavior under complex loading. (Shahzamanian et al. 2021) contribute further by using the cohesive zone model (CZM) and extended finite element method (XFEM) to analyze fiber-matrix debonding and matrix cracking in composite materials, providing valuable insights into the material degradation in response to crack propagation. These studies collectively demonstrate the importance of integrating experimental validation with numerical models for enhanced predictive accuracy in crack load and stress intensity assessments.

Another prominent trend in recent research is the focus on slow crack propagation in heterogeneous materials, as illustrated by (Kierfeld and Vinokur 2006), who investigate quasi-static crack growth under arbitrary pressure distributions. By employing surface and volume integral equations combined with fast Fourier transform techniques, they construct equilibrium crack shapes in layered elastic media. This approach is particularly relevant for layered

materials, where crack behavior varies significantly due to differences in elastic properties and fracture toughness across layers. Gao et al. (2018) also explore crack behavior in heterogeneous settings, specifically addressing hot cracking in laser-welded TRIP steel. Their thermal-mechanical finite element (FE) model, validated through experimental data, examines temperature and strain evolution in the weld mushy zone, identifying a critical strain threshold for crack initiation. (Paul 2021) further investigate aggregate interlocking in cracked concrete using numerical models, addressing the complex shear and normal stress transfer across open cracks. Together, these studies highlight the necessity for numerical methods that can accommodate the distinct material characteristics and stress distribution in layered and composite materials, ensuring reliable predictions of crack behavior under varied environmental conditions.

Advancements in computational approaches to fatigue and crack propagation under thermal and mechanical loads further enrich the literature on numerical crack analysis. (Wang et al. 2023) introduce a Bézier-based extended isogeometric analysis (XIGA) model, which accurately predicts crack propagation rates in plates under welding residual stress and thermal loads without requiring re-meshing. This innovative approach leverages discontinuous enrichment functions for more precise stress intensity factor calculations in regions affected by thermal and residual stress. (Bergara et al. 2020) complement these findings with a focus on hot cracking thresholds in laser welding, revealing critical strain values that predict the onset of cracks in welded materials. Similarly, (Liu et al. 2018) use XFEM to simulate matrix cracking in composite materials, providing a numerical framework to understand material degradation under cyclic loading. These studies underscore a growing emphasis on developing specialized numerical methods that accurately predict crack initiation and propagation in response to cyclic and thermal stress, a critical area of interest in structural engineering.

Despite the advancements in numerical methods, several research gaps persist. (Y. Ji et al. 2024) method for predicting first crack load in concrete beams, while accurate, may need further exploration for varied loading conditions and different structural shapes. (Faridi et al. 2024) algorithms for stress intensity factors perform well in homogeneous and heterogeneous media but could benefit from extensions that incorporate complex material configurations, such as composites with multiple interface layers. (Fartash, Ayatollahi, and Bagheri 2019; Ge et al. 2016) address crack propagation in heterogeneous and welded materials, respectively, yet there remains a need for universally adaptable methods capable of handling diverse material compositions and stress environments. The current methods tend to be material-specific, highlighting a critical gap for future research aimed at creating multi-functional tools that integrate different numerical approaches, such as XFEM, CZM, and XIGA, into cohesive models that can handle varied materials and conditions with precision.

Previous studies, including XFEM, NMM, and Integral methods (IEIM) by (Scamardo et al. 2022), have demonstrated significant improvements in computational efficiency and accuracy. However, several critical issues remain. For example, Scamardo's non-standard finite element method is effective for tensile cracks in quasi-brittle materials, but its performance in complex materials or mixed-mode cracks has not been thoroughly tested. Similarly, although XFEM combined with IEIM effectively handles thermal shock cracks in composite materials, the method's adaptability to broader and more complex scenarios remains limited. Additionally, hybrid methods like Physics-Informed Neural Networks (PINNs) proposed by (Gu et al. 2023), despite their accuracy, still face challenges with scalability for industrial-sized applications and

have material-specific limitations. Thus, the main research gap is the absence of universally adaptable numerical methods capable of efficiently addressing multiple crack interactions across diverse material types and loading conditions, highlighting the need for further experimental validation and development of integrated modeling approaches.

In conclusion, the reviewed studies illustrate substantial progress in numerical methods for crack detection and analysis, with each study contributing unique methodologies tailored to specific structural and material challenges. (Chen et al. 2020; Liu et al. 2021; Lv et al. 2020; Scamardo et al. 2022) offer specialized advancements that pave the way for more comprehensive and adaptable crack analysis tools. These developments emphasize computational efficiency and precision, as evidenced in the works of (Kozlov, Dolganov, and Slobodin 2024; Shoheib et al. 2022; Wang et al. 2019), which highlight energy-efficient, multi-functional methods for complex structural issues. Studies by (Markov and Kanaun 2018; Orie and Ogbonna 2024; Tikhomirov 2020) further showcase significant strides in accurately predicting stress intensity and crack propagation across diverse material interfaces and structural complexities. Despite these achievements, there remains a strong impetus for further research to create universally applicable models that address current limitations in scalability, computational demand, and multi-material adaptability. Such advancements would bridge existing knowledge gaps, enhancing the robustness and versatility of numerical methods in crack detection and structural analysis (Kang et al. 2022).

Research Question

Research questions are crucial in a systematic literature review (SLR) because they provide the foundation and direction for the entire review process. They guide the scope and focus of the SLR, helping to determine which studies to include or exclude, ensuring that the review remains relevant and specific to the topic of interest. A well-defined research question ensures that the literature search is exhaustive and systematic, covering all relevant studies that address key aspects of the topic. This minimizes the risk of bias and ensures a complete overview of the existing evidence. Additionally, research questions facilitate the categorization and organization of data from included studies, providing a framework for analyzing findings and synthesizing results to draw meaningful conclusions. They also enhance clarity and focus, avoiding ambiguity and keeping the review concentrated on specific issues, making the findings more actionable and relevant. Furthermore, well-formulated research questions contribute to the transparency and reproducibility of the review, allowing other researchers to follow the same process to verify findings or extend the review to related areas. Ultimately, research questions ensure that the review aligns with the overall objectives of the study, whether it is to identify gaps in the literature, evaluate the effectiveness of interventions, or explore trends in a specific field, making them the backbone of a rigorous, focused, and relevant systematic literature review.

Specifying the Research Questions (RQs) is the most important activity at the planning stage but also the most important part of any SLR, because it drives the entire review methodology (Mangaroo-Pillay and Coetzee 2022). Considering that the goal of our SLR is to identify and analyze the state-of-the-art in. The PICo framework is a mnemonic style used to formulate research questions, particularly in qualitative research proposed by (Adams and Smith 2003) was applied in this study. PICo stands for Population, Interest, and Context. Using the PICo framework helps in structuring research questions clearly and systematically by breaking down the key elements of the study into these three components. This approach ensures that the

research is focused and the questions are well-defined, making it easier to search for relevant literature or design a study. This study achieved there research question as below;

- i. How effective are current numerical methods in detecting and analyzing cracks in large-scale infrastructure projects, and what improvements can be made to enhance their accuracy and reliability?
- ii. What is the impact of dynamic loading conditions on crack propagation in high-stress environments, and how can dynamic analysis techniques be optimized to predict and mitigate crack growth?
- iii. How can advanced fracture analysis techniques be tailored to specific materials to improve their fracture resistance, and what role do these techniques play in the development of new materials with enhanced durability?

Material and Methods

For conducting systematic literature reviews, the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) approach is a widely accepted standard that guarantees transparency, completeness, and consistency throughout the procedure by (Saritaş and Topraklıkoğlu 2022). Researchers can improve the accuracy and rigour of their analysis by adhering to PRISMA guidelines, which provide guidance on how to systematically identify, screen, and include studies in their review. The method also highlights the significance of randomised studies, acknowledging their ability to lessen bias and provide strong evidence for the review. Two important databases, Web of Science and Scopus, were used in this analysis because of their wide coverage and robustness.

The PRISMA approach is organized into four key stages: identification, screening, eligibility, and data abstraction. In the identification phase, databases are searched to locate all relevant studies. The screening phase then involves evaluating these studies against predefined criteria to eliminate irrelevant or low-quality research. During the eligibility phase, the remaining studies are thoroughly assessed to confirm they meet the inclusion criteria. Finally, data abstraction focuses on extracting and synthesizing data from the included studies, which is essential for deriving meaningful and reliable conclusions. This structured method ensures that the systematic review is conducted with rigor, leading to trustworthy results that can guide future research and practice.

Identification

In a Systematic Literature Review (SLR), the identification phase is critical as it sets the foundation for gathering relevant research. Using databases like Scopus and Web of Science (WoS) allows researchers to capture a comprehensive set of records related to specific topics. In this case, keywords such as "technology," "Numerical Methods," and "Crack material" were employed to locate studies relevant to crack detection, material analysis, and the application of numerical methods in these contexts. Through this process, 478 records were identified from Scopus, and 32 were sourced from WoS, totaling 510 records.

The inclusion of Scopus and WoS in the identification process provides several advantages. Scopus, known for its extensive indexing of peer-reviewed research across numerous fields, enables researchers to access a wide range of engineering, materials science, and applied technology studies. Web of Science, with its focus on high-impact and quality journals, complements Scopus by ensuring the inclusion of seminal and influential works in the dataset.

Together, these databases enhance the diversity and quality of the literature included in the SLR, ensuring that the review is both broad and rigorous.

Selecting appropriate keywords is another important aspect of the identification phase. Keywords like "technology," "Numerical Methods," and "Crack material" are carefully chosen to align with the research objectives, facilitating the discovery of studies focused on applying computational and analytical techniques in material crack analysis. By narrowing the focus to these keywords, the identification phase remains aligned with the specific objectives of exploring technological advancements and numerical methods applied to crack analysis in various materials.

In conclusion, the identification phase of an SLR using Scopus and WoS databases has yielded a substantial set of records. This systematic approach not only broadens the understanding of numerical methods and technology applications in material crack analysis but also lays the groundwork for further stages of the review, including screening, eligibility *assessment*, and *synthesis*.

Table 1: The Search String.

Scopus	TITLE-ABS-KEY ("Numerical Method" AND (crack* OR "crack material") AND ("Crack Detection" OR "Crack Analysis")) AND (LIMIT-TO (PUBYEAR , 2023) OR LIMIT-TO (PUBYEAR , 2024)) AND (LIMIT-TO (DOCTYPE , "ar")) AND (LIMIT-TO (PUBSTAGE , "final")) AND (LIMIT-TO (SRCTYPE , "j")) AND (LIMIT-TO (LANGUAGE , "English")) AND (LIMIT-TO (SUBJAREA , "MATH"))
Date of Access: January 2025	
WoS	"Numerical Method" AND (crack OR "crack material") AND ("Crack Detection" OR "Crack Analysis") (Topic) and 2024 (Publication Years) and Article (Document Types) and English (Languages)
Date of Access: January 2025	

Screening

During the screening phase, the collection of potentially relevant research items is evaluated for alignment with the predefined research question(s). Common content-related criteria used in this phase include the selection of studies focused on "Numerical Methods in Crack Detection and Analysis." At this step, duplicate papers are removed from the list of collected items. The initial stage of screening resulted in the exclusion of 510 publications, while the second stage closely examined 20 papers based on various inclusion and exclusion criteria (see Table 2). The primary criterion for inclusion was research literature, as it serves as the main source of practical insights. This includes reviews, meta-syntheses, meta-analyses, books, book series, chapters, and conference proceedings, excluding those covered in the latest study. Additionally, the review was limited to English-language publications, focusing exclusively on

materials published between 2023 and 2024. Ultimately, non-research publications were excluded based on duplication criteria.

Table 2: The Selection Criterion Is Searching

Criterion	Inclusion	Exclusion
Language	English	Non-English
Time line	2023 – 2024	< 2023
Literature type	Journal (Article)	Conference, Book, Review
Publication Stage	Final	In Press
Country	All	Besides All
Subject	Mathematics	Besides Mathematics

Eligibility

The final review sample was formed after applying all inclusion and exclusion criteria. It is essential to provide a complete list of the research items in this sample to clarify the foundation of the study's results for readers. In the eligibility stage, a total of 20 articles were reviewed. At this stage, each article's title and key content were meticulously examined to confirm they met the inclusion criteria and aligned with the study's research objectives. Consequently, 5 publications were excluded due to a lack of significant relevance to the study's purpose based on empirical data. This process resulted in 15 papers being selected for detailed evaluation (see Figure 1).

Data Abstraction and Analysis

An integrative analysis was employed as an assessment strategy in this study to examine and synthesize various research designs, specifically quantitative methods. The primary objective was to identify relevant topics and subtopics. Data collection marked the initial step in developing the study's theme. As shown in Figure 2, the authors carefully analyzed a selection of 15 publications for content pertinent to the study's topics. The next phase involved assessing notable studies related to Numerical Methods in Crack Material Detection and Analysis, focusing on the methodologies and findings presented. The lead author worked with co-authors to develop themes based on evidence within the study's framework. A log was maintained throughout the data analysis process to capture analyses, perspectives, challenges, or other relevant insights for data interpretation. Finally, the authors compared results to check for any inconsistencies in the theme design process. Any conceptual disagreements among the authors were discussed collaboratively to ensure alignment.

Table 3: Number and details of Primary Studies Database

No	Authors	Title	Year	Journal	Scopus	Web of Science
1	Su C et al.	A SFBEM–FEM coupling method for solving crack problems based on Erdogan fundamental solutions	2023	Journal Of Engineering Mathematics	/	
2	Li W.; et al.	An explicit improved meshless numerical manifold method for dynamic crack propagation	2024	Theoretical And Applied Fracture Mechanics	/	
3	Wang Z.; et al	Micro-crack in solids evaluation based on zero-frequency component of the critically refracted longitudinal wave	2024	Measurement Science And Technology	/	
4	Li K.; Wang F.	A combined displacement discontinuity-interaction integral method for computing stress intensity factors and T-stress	2024	International Journal Of Solids And Structures	/	
5	Qu W.; et al.	A stable numerical framework for long-time dynamic crack analysis	2024	International Journal Of Solids And Structures	/	
6	Ju B.; et al.	A generalized finite difference method for 2D dynamic crack analysis	2024	Results In Applied Mathematics	/	
7	Gu Y.; et al.	Enriched physics-informed neural networks for 2D in-plane crack analysis: Theory and MATLAB code	2023	International Journal Of Solids And Structures	/	
8	Kushwaha N. et al.	Nonlinear dynamic analysis of two-disk rotor system containing an unbalance influenced transverse crack	2023	Nonlinear Dynamics	/	
9	Guo Z.; et al.	Superiority of eigen COD boundary integral equations in simulating multiple crack problems in linear elastic solids	2024	Theoretical And Applied Fracture Mechanics	/	
10	Kalay O.C.; et al. [22]	Effects of tooth root cracks on vibration and dynamic transmission error responses of asymmetric gears: A comparative study	2024	Mechanics Based Design Of Structures And Machines	/	

11	Ji W.; et al. [23]	Dynamic analysis of cracked pipe elbows: Numerical and experimental studies	2024	International Journal Of Mechanical Sciences	/	
12	Azinpour E.; et al. [24]	Phase-field ductile fracture analysis of multi-materials and functionally graded composites through numerical and experimental methods	2023	Theoretical And Applied Fracture Mechanics	/	
13	Mutra, RR; et al. [25]	Crack fault diagnosis in rotor bearing system by transient and study state time domain analysis	2025	Measurement		/
14	Hu, C; et al. [26]	A phase-field fatigue fracture model considering the thickness effect	2024	Engineering Fracture Mechanics		/
15	Faridi, MA; et al. [27]	Damage quantification in beam-type structures using modal curvature ratio	2024	Innovative Infrastructure Solutions		/

Quality of Appraisal

Following the guidelines outlined by (Khoie, Mohammadi, and Yeghaneh 2023) once we identified the primary studies (PS), we proceeded to assess the quality of the research presented and quantitatively compare them. In this study, we adopted the quality assessment (Abouzahra, Sabraoui, and Afdel 2020) which includes six quality assessment criteria (QAs) for our systematic literature review (SLR). The scoring process for each criterion offers three ratings: "Yes" (Y), scored at 1 if fully met; "Partly" (P), scored at 0.5 if partially met with some gaps; and "No" (N), scored at 0 if not met at all.

1. QA1. Is the purpose of the study clearly stated?
2. QA2. Is the interest and the usefulness of the work clearly presented?
3. QA3. Is the study methodology clearly established?
4. QA4. Are the concepts of the approach clearly defined?
5. QA5. Is the work compared and measured with other similar work?
6. QA6. Are the limitations of the work clearly mentioned?

Each expert independently evaluates the study based on these criteria, and their scores are then combined to calculate the overall mark. To qualify for the next stage, a study must achieve a total score exceeding 3.0 across the assessments of all three experts. This threshold ensures that only studies meeting a defined quality standard progress further.

In this systematic literature review, the quality assessment of selected studies was performed rigorously based on clearly defined criteria adapted from Anas Abouzahra et al. (2020), following guidelines established by Kitchenham and Charters (2007). Each primary study was independently evaluated by three experts according to six specific criteria: (1) clarity of study objectives, (2) usefulness and relevance of findings, (3) clarity and robustness of methodology, (4) clear definition of concepts used, (5) comparison with existing works, and (6) explicit mention of limitations. The quality assessment was quantified using a scoring system of ‘Yes’ (1 point), ‘Partly’ (0.5 points), and ‘No’ (0 points). Only studies achieving a cumulative score above 3.0 were included for detailed analysis, ensuring that the review maintained high methodological rigor and reliability.

Result and Finding

Background of selected study: based on quality assessment, table 4 shown the result of assessment performance for selected primary studies. The table presents a dataset of 15 problem sets (PS1 to PS15), each evaluated across six questions (QA1 to QA6), with scores ranging from 0 to 1 for each question. The "Total Mark" column sums the scores for each problem set, while the "Percentage (%)" column calculates the percentage score based on the total possible score of 6. This data provides insights into the performance across different problem sets, highlighting variations in total marks and percentages, which can be used to assess overall performance and identify areas for improvement. Here is the quality assessment table for the selected papers:

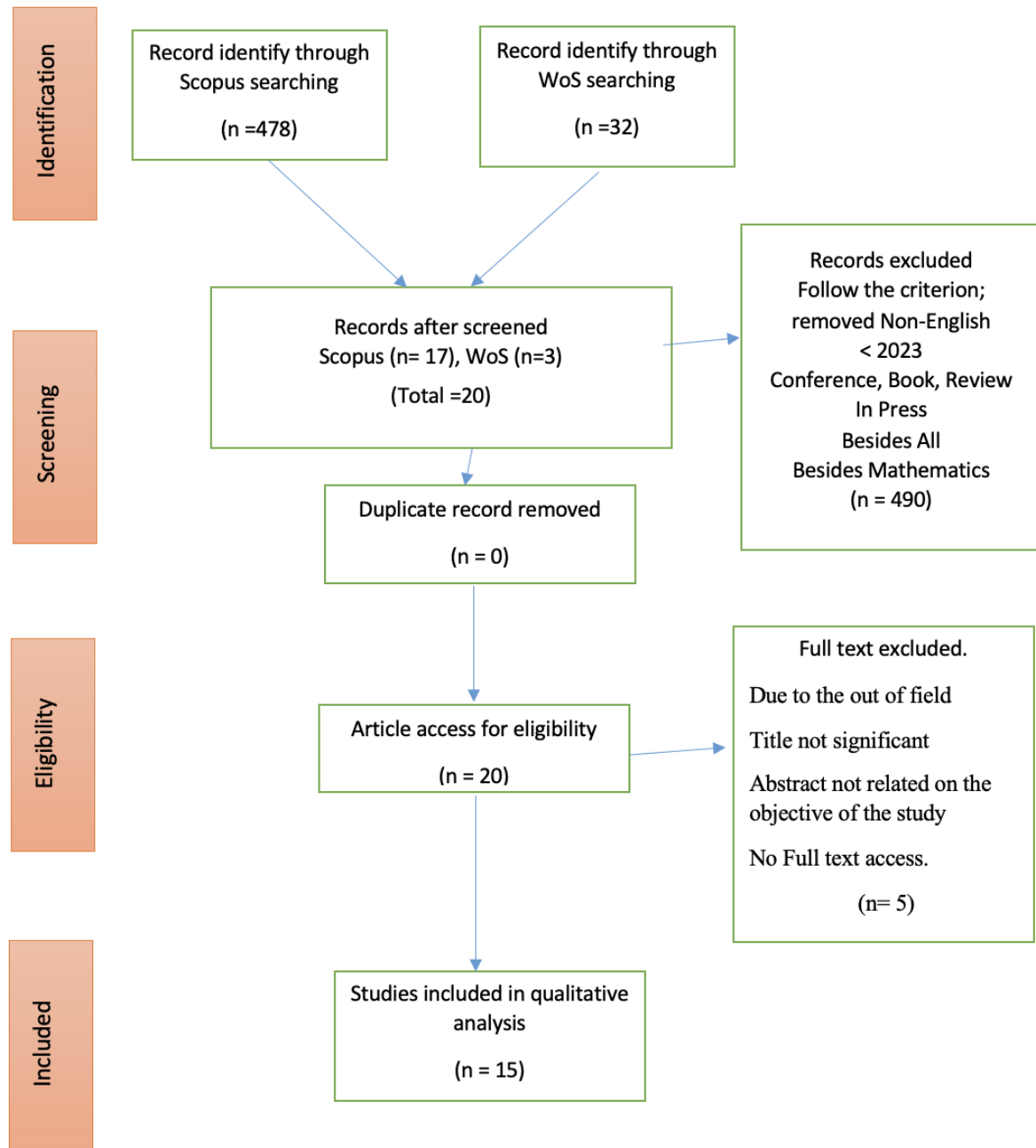


Figure 1: Flow Diagram Of The Proposed Searching Study (Khoie et al. 2023)

Table 4: Assessment Performance for Selected Primary Studies

Data	QA1	QA2	QA3	QA4	QA5	QA6	Total Mark	Percentage (%)
PS1	1	1	1	1	0.5	0.5	5	83.33%
PS2	1	1	1	1	1	0.5	5.5	91.67%
PS3	1	1	1	1	0	0.5	4.5	75.00%
PS4	1	1	1	1	1	0.5	5.5	91.67%
PS5	1	1	1	1	1	1	6	100%
PS6	1	1	1	1	0.5	0.5	5	83.33%
PS7	1	1	1	1	1	0.5	5.5	91.67%
PS8	1	1	1	1	1	0.5	5.5	91.67%
PS9	1	1	1	1	1	1	6	100%
PS10	1	1	1	1	0.5	0.5	5	83.33%
PS11	1	1	1	1	0.5	0.5	5	83.33%
PS12	1	1	1	1	1	0.5	5.5	91.67%
PS13	1	1	0.5	1	0.5	0	4.0	66.67%
PS14	1	0.5	1	1	1	1	5.5	91.67%
PS15	1	1	1	0.5	1	0.5	5.0	83.33%

Numerical Methods for Crack Detection and Analysis (CDA)

The findings from recent studies on numerical methods for crack detection and analysis demonstrate substantial advancements in accurately identifying stress and fracture behaviours within materials. For instance, Su, Cai, and Xu [36] introduced an SFBEM-FEM coupling method that integrates the boundary element method (BEM) with the finite element method (FEM), focusing on complex crack patterns, particularly in materials like steel anchorage boxes used in suspension bridges. Their results reveal a significant improvement in computational efficiency and precision in determining stress intensity factors (SIFs) for multi-crack scenarios, highlighting the method's potential for real-world applications requiring high accuracy in structural integrity assessment. Similarly, Guo, Ren, and He advanced (Zhang and Ge 2008) advanced the modeling of multi-crack simulations in two-dimensional solids by employing eigen crack opening displacement (COD) boundary integral equations. Their method, benchmarked against the fast multipole boundary element method (BEM), provides highly accurate results even for thousands of cracks, showing superior computational efficiency that allows complex simulations to be performed on standard computers, significantly broadening the accessibility of such analyses.

Furthering the capabilities in dynamic crack analysis, (Zedan Khalel, Khan, and Starr 2023) developed the Improved Meshless Numerical Manifold Method (iMNNM), specifically designed to simulate dynamic crack propagation under various stress conditions. This approach incorporates an energy-conserving degrees-of-freedom inheritance strategy and a novel mass lumping method, which together enable the method to maintain stable time-stepping across simulations. The study's results underscore the method's effectiveness in simulating complex dynamic fractures in rock materials, particularly useful for geomechanical applications where accurate crack propagation modeling under dynamic loading is essential. Moreover, (Wang et al. 2022) explored the Generalized Finite Difference Method (GFDM) for transient elastodynamic crack analysis, utilizing a local Taylor series and moving-least square approximation. Their method efficiently captures stress intensity factors (SIFs) with high accuracy and stability under variable loading conditions, reinforcing its suitability for applications that require a high degree of precision in time-sensitive fracture scenarios.

(Shlyannikov, Fedotova, and Khamidullin 2023) contributed to mixed-mode crack analysis with a combined displacement discontinuity-interaction integral approach, allowing for simultaneous evaluation of SIFs and T-stress without decomposing elastic fields. By integrating auxiliary fields into the displacement discontinuity method, this approach successfully handles mixed-mode problems and provides explicit expressions for displacement gradients, ensuring accuracy in calculating fracture parameters in complex geometries. When evaluated through numerical examples, this combined method proved robust for two-dimensional crack analysis, providing valuable insights into mixed-mode fractures where traditional methods fall short.

The cumulative research on crack detection and analysis methods emphasizes an ongoing trend toward enhancing computational efficiency, accuracy, and adaptability across varied applications. Each study introduces specific improvements, from Su et al.'s SFBEM-FEM coupling method's high precision in structural applications to Guo et al.'s eigen COD BIEs that facilitate large-scale simulations. The advancements are particularly notable in areas requiring complex crack analysis, such as structural engineering, geomechanics, and materials science, and offer significant implications for designing reliable, cost-effective analysis tools that accommodate increasingly complex fracture problems.

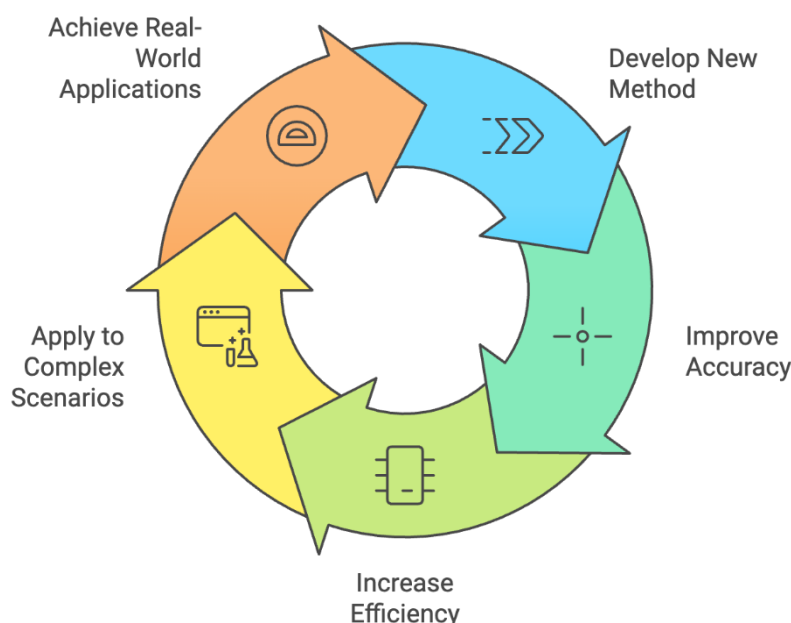


Figure 2: Cycle of Numerical Method Advancements

Dynamic Analysis and Crack Propagation (CAP)

Research in dynamic analysis and crack propagation offers significant insights into crack detection methods across different mechanical structures, including gears, rotors, and fluid-conveying pipes. (Mohammed and Rantatalo 2016) developed a stable long-time framework for dynamic crack analysis, combining the Krylov deferred correction method for temporal discretization and the generalized finite difference method (GFDM) for spatial partial differential equations. Their model is particularly effective in minimizing boundary condition errors and stabilizing large time-step simulations, with validation tests showing precise stress and displacement field results under diverse impact loadings. This approach holds promise for extended-duration crack propagation analyses, offering a stable alternative to traditional mesh-dependent techniques. (Païdoussis 2022) contributed to fluid-conveying pipe elbow dynamics by integrating spectral and finite element methods, enabling accurate simulation of fluid-structure interactions and crack breathing effects. Their model captures frequency veering and vibration mode switching based on elbow bending parameters, making it an effective diagnostic tool for assessing crack behavior in fluid systems through stress response harmonics.

Dynamic analysis methods applied to rotating machinery and gear systems further underscore the importance of vibration-based fault detection for early crack identification. (Hossain and Wu 2018) investigated the vibrational responses of a cracked two-disk rotor system, observing that changes in natural frequencies and whirl orbit patterns are highly sensitive to transverse cracks. Their model, which utilizes the finite element method (FEM) and Newmark time integration, shows that the rotor's whirl orbit shapes can serve as reliable indicators of both crack depth and location, with sensitivity increasing near critical speeds. (Hotait and Kahraman 2013) examined gear root cracks and found that asymmetric gears demonstrate distinct dynamic responses influenced by drive-side pressure angles, enhancing the accuracy of early fault detection. Their study utilized dynamic transmission error (DTE) and vibration analysis to determine that vibration signals, influenced by gear tooth asymmetry, offer superior diagnostic precision over DTE alone in detecting tooth root faults.

Rotor-bearing systems, as analyzed by (Ri et al. 2022), showcase the effectiveness of transient and steady-state time domain analyses in detecting transverse fatigue cracks, which can lead to severe failures if unmonitored. By integrating the Newton-Raphson, Houbolt, and Harmonic Balance methods, this research assesses orbital patterns and FFT responses under various load conditions, achieving a high correlation between FEM and experimental data for crack-induced vibrations. The study models both open and breathing wedge cracks, highlighting the importance of redundant diagnostic methods to enhance reliability. These findings not only validate the effectiveness of FEM in transient crack detection but also establish a robust framework for real-time monitoring in critical rotating machinery applications.

The advancements presented in dynamic analysis and crack propagation reinforce the value of combining numerical methods with experimental validation, particularly in systems exposed to complex dynamic stresses. Techniques such as those presented by Qu et al., Ji et al., and Kushwaha et al. underscore the critical role of frequency and vibration response analysis in accurately detecting and predicting crack behavior. These methods contribute significantly to safer and more efficient structural health monitoring in sectors reliant on rotating machinery, fluid systems, and gear mechanisms.

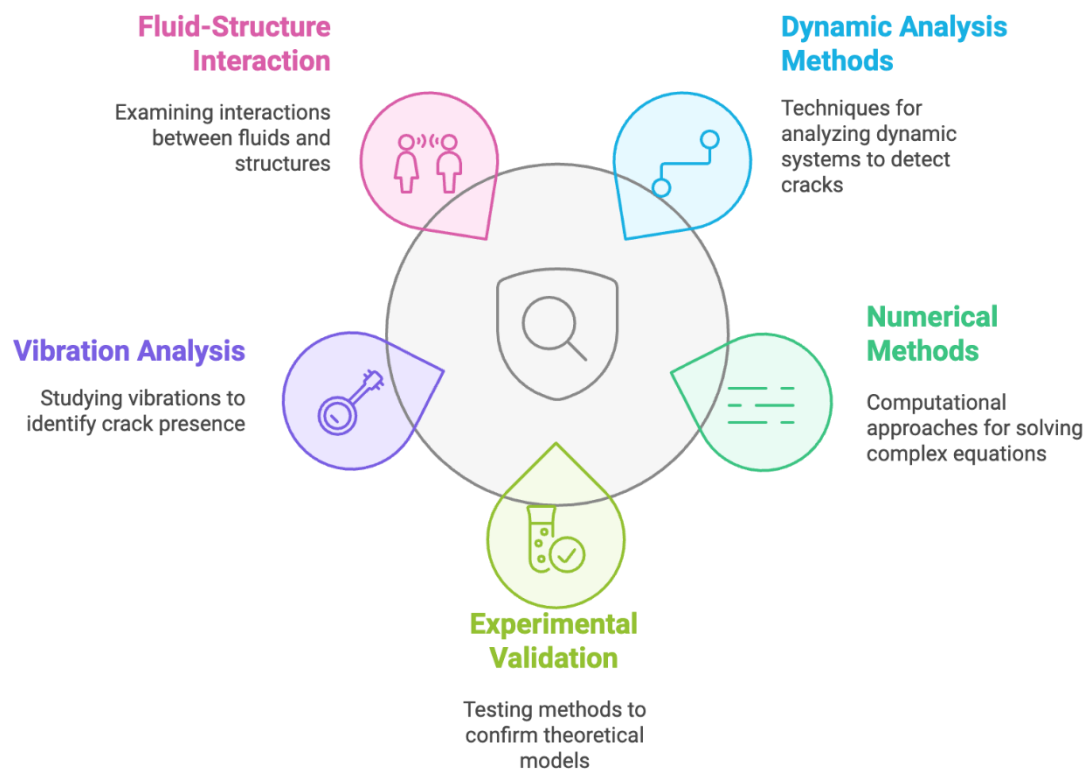


Figure 3: Advancements in Crack Detection

Material-Specific and Advanced Fracture Analysis Techniques (MAFA)

Recent studies on material-specific and advanced fracture analysis techniques provide substantial progress in the detection and quantification of micro-cracks and other fracture behaviors in complex materials. (Zhao et al. 2018) explored the detection of micro-cracks in solids by employing a zero-frequency component generated by critically refracted longitudinal

(LCR) waves. This approach leverages the distortion of ultrasonic waves in micro-cracked regions to reveal a zero-frequency component, which was found to be more sensitive to micro-crack detection compared to traditional second and third harmonics. Through numerical simulations and experiments, the study demonstrated that this technique offers enhanced detection capability for micro-cracks, particularly in engineering materials susceptible to subtle damage. (Gault et al. 2023) introduced the modal curvature ratio method for damage quantification in beam-type structures, proving its effectiveness in locating and measuring structural damage with minimal information on the structure's complete model. The method displayed high accuracy in identifying stiffness loss even as low as 5%, marking a significant advancement in non-invasive structural health monitoring, particularly for real-world applications where beam-type elements are prevalent.

Advanced phase-field approaches provide a notable direction in fracture analysis for multi-materials and functionally graded materials (FGMs). (Shahzamanian, Partovi, and Wu 2020) investigated ductile fracture behaviors in FGMs through a phase-field approach that integrates an elastoplastic material framework with damage-driving forces. This framework effectively captures the complex fracture patterns and resistance shifts induced by material property mismatches within FGMs, particularly during crack propagation. The study's experimental validations using materials such as 316L and IN718 demonstrate that the model effectively predicts fracture behavior under real-world conditions, affirming its applicability to composite and gradient materials. (Kushwaha and Patel 2023) addressed the challenges of variable-thickness geometries in aero-engine turbine structures by developing a modified phase-field fatigue fracture model that incorporates thickness-related fracture toughness variations (Gc-B curve). This adaptation enables accurate fracture toughness predictions across different thicknesses, which were validated through close alignment with experimental data. Such advancements highlight the model's potential for enhancing the safety design and life prediction of aerospace components.

Physics-informed neural networks (PINNs) represent a significant breakthrough in meshless fracture analysis, enabling efficient analysis of two-dimensional in-plane cracks without the complexities associated with traditional meshing methods. (Orie and Ogbonna 2024) applied an enriched PINNs-based model for 2D crack analysis, embedding asymptotic functions to model near-tip displacement and stress fields with precision. By satisfying governing equations and boundary conditions directly, this approach bypasses nodal refinement and is computationally lighter than conventional finite element or boundary element methods. Testing on various crack configurations demonstrated that the PINNs method reliably computes stress intensity factors (SIFs), achieving a high degree of accuracy with fewer computational resources. The availability of supplementary MATLAB code and datasets further enhances its accessibility and adoption in fracture mechanics research, offering a simplified yet effective tool for crack analysis.

These advancements in fracture analysis reveal a clear trend toward developing efficient, accurate, and adaptable techniques for diverse materials and structural forms. From the zero-frequency LCR wave component method for micro-cracks to the phase-field model's adaptability to composite materials, each approach addresses unique challenges in the detection and prediction of crack behaviors. The continued integration of machine learning and physics-informed modeling, as seen with PINNs, highlights an evolution toward methodologies that

optimize both computational efficiency and diagnostic accuracy, paving the way for real-time, reliable applications in structural health monitoring and materials science.

Discussion and Conclusion

Recent advancements in numerical methods for crack detection and analysis have significantly improved modeling stress and fracture behaviors across diverse materials and structural applications. Techniques such as the hybrid boundary element-finite element method (BEM-FEM) have enhanced stress intensity factor (SIF) calculations, demonstrating effectiveness in complex structural components like suspension bridge anchorage boxes. Similarly, the eigen crack opening displacement (COD) boundary integral equations method effectively simulates thousands of cracks efficiently on standard computational resources, significantly broadening its applicability. Other innovative methods like the Improved Meshless Numerical Manifold Method (iMNM) and the Generalized Finite Difference Method (GFDM) provide stable and precise dynamic crack propagation analyses, significantly benefiting geomechanical contexts and other dynamically loaded structures.

Despite these advancements, several limitations persist. One major challenge is the computational intensity and limited scalability when numerical methods are applied to large-scale industrial structures, as current techniques often demand extensive computational resources. Additionally, the accuracy of these numerical predictions remains highly dependent on material-specific parameters, reducing their generalizability across various materials and structural conditions. Furthermore, most studies reviewed have focused on specialized cases or single-crack scenarios, highlighting a crucial gap in universally adaptable numerical models capable of handling multi-crack interactions effectively across different materials and loading conditions.

Therefore, future research should prioritize developing more universally adaptable numerical models that integrate various techniques, such as hybrid approaches combining traditional numerical methods with artificial intelligence, to optimize computational efficiency and improve predictive accuracy. Moreover, extensive experimental validations across diverse material types and structural configurations are necessary to enhance these numerical methods' robustness, reliability, and applicability in real-world scenarios. Such developments would bridge existing knowledge gaps, enabling the creation of comprehensive and reliable structural health monitoring systems for safer and more efficient engineering practices.

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Reference

- Abouzahra, Anas, Ayoub Sabraoui, and Karim Afdel. 2020. "Model Composition in Model Driven Engineering: A Systematic Literature Review." *Information and Software Technology* 125.
- Adams, Jon, and T. Smith. 2003. "Qualitative Methods in Radiography Research: A Proposed Framework." *Radiography* 9(3). doi:10.1016/S1078-8174(03)00061-0.
- Azinpour, E., S. Rzepa, D. Melzer, A. Reis, J. Džugan, and J. M. A. Cesar de Sa. 2023. "Phase-Field Ductile Fracture Analysis of Multi-Materials and Functionally Graded

- Composites through Numerical and Experimental Methods.” *Theoretical and Applied Fracture Mechanics* 125. doi:10.1016/j.tafmec.2023.103906.
- Bergara, A., A. Arredondo, J. Altuzarra, and J. M. Martínez-Esnaola. 2020. “Calculation of Stress Intensity Factors in Offshore Mooring Chains.” *Ocean Engineering* 214. doi:10.1016/j.oceaneng.2020.107762.
- Chai, Hao, Jun Lv, and Yumei Bao. 2020. “Numerical Solutions of Hypersingular Integral Equations for Stress Intensity Factors of Planar Embedded Interface Cracks and Their Correlations with Bimaterial Parameters.” *International Journal of Solids and Structures* 202. doi:10.1016/j.ijsolstr.2020.06.014.
- Chen, Yi Jao, Yong Shan Lai, and Yen Han Lin. 2020. “BIM-Based Augmented Reality Inspection and Maintenance of Fire Safety Equipment.” *Automation in Construction* 110. doi:10.1016/j.autcon.2019.103041.
- Faridi, Md Arif, Koushik Roy, and Vaibhav Singhal. 2024. “Damage Quantification in Beam-Type Structures Using Modal Curvature Ratio.” *Innovative Infrastructure Solutions* 9(2). doi:10.1007/s41062-023-01353-w.
- Fartash, A. H., M. Ayatollahi, and R. Bagheri. 2019. “Transient Response of Dissimilar Piezoelectric Layers with Multiple Interacting Interface Cracks.” *Applied Mathematical Modelling* 66. doi:10.1016/j.apm.2018.09.030.
- Gault, Baptiste, Heena Khanchandani, Thoudeden Sukumar Prithiv, Stoichko Antonov, and T. Ben Britton. 2023. “Transmission Kikuchi Diffraction Mapping Induces Structural Damage in Atom Probe Specimens.” *Microscopy and Microanalysis* 29(3). doi:10.1093/micmic/ozad029.
- Ge, Huibin, Bin Zhang, Xiaomin Gu, Haojie Liang, Huimin Yang, Zhe Gao, Jianguo Wang, and Yong Qin. 2016. “A Tandem Catalyst with Multiple Metal Oxide Interfaces Produced by Atomic Layer Deposition.” *Angewandte Chemie - International Edition* 55(25). doi:10.1002/anie.201600799.
- Gu, Y., C. Zhang, P. Zhang, M. V. Golub, and B. Yu. 2023. “Enriched Physics-Informed Neural Networks for 2D in-Plane Crack Analysis: Theory and MATLAB Code.” *International Journal of Solids and Structures* 276. doi:10.1016/j.ijsolstr.2023.112321.
- Guo, Z., X. Ren, and D. He. 2024. “Superiority of Eigen COD Boundary Integral Equations in Simulating Multiple Crack Problems in Linear Elastic Solids.” *Theoretical and Applied Fracture Mechanics* 133. doi:10.1016/j.tafmec.2024.104569.
- Hossain, Mobarak, and Helen Wu. 2018. “Crack Breathing Behavior of Unbalanced Rotor System: A Quasi-Static Numerical Analysis.” *Journal of Vibroengineering* 20(3). doi:10.21595/jve.2018.19692.
- Hotait, M. A., and A. Kahraman. 2013. “Experiments on the Relationship between the Dynamic Transmission Error and the Dynamic Stress Factor of Spur Gear Pairs.” *Mechanism and Machine Theory* 70. doi:10.1016/j.mechmachtheory.2013.07.006.
- Hu, Chun, Hongyu Qi, Shaolin Li, Xiaoguang Yang, and Duoqi Shi. 2024. “A Phase-Field Fatigue Fracture Model Considering the Thickness Effect.” *Engineering Fracture Mechanics* 296. doi:10.1016/j.engfracmech.2024.109855.
- Ji, W., H. Ma, F. Liu, W. Sun, and D. Wang. 2024. “Dynamic Analysis of Cracked Pipe Elbows: Numerical and Experimental Studies.” *International Journal of Mechanical Sciences* 281. doi:10.1016/j.ijmecsci.2024.109580.
- Ji, Yanting, Sheng Sun, Aijiu Chen, Fen Yang, Shihua Bai, and Xiaoyan Han. 2024. “Study on Crack Resistance and Calculation Model of RAC Beams Strengthened with Prestressed CFRP.” *International Journal of Concrete Structures and Materials* 18(1). doi:10.1186/s40069-023-00638-9.

- Ju, B., B. Yu, and Z. Zhou. 2024. "A Generalized Finite Difference Method for 2D Dynamic Crack Analysis." *Results in Applied Mathematics* 21. doi:10.1016/j.rinam.2023.100418.
- Kalay, O. C., O. Doğan, C. Yuce, and F. Karpat. 2024. "Effects of Tooth Root Cracks on Vibration and Dynamic Transmission Error Responses of Asymmetric Gears: A Comparative Study." *Mechanics Based Design of Structures and Machines* 52(5):2569–2604. doi:10.1080/15397734.2023.2186892.
- Kang, Ge, Youjun Ning, Pengwan Chen, Siping Pang, and Yongbo Shao. 2022. "Comprehensive Simulations of Rock Fracturing with Pre-Existing Cracks by the Numerical Manifold Method." *Acta Geotechnica* 17(3). doi:10.1007/s11440-021-01252-3.
- Khoie, Mahdiah, Mahdi Mohammadi, and Mehri Ezadi Yeghaneh. 2023. "Scientific Social Networks in the Mirror of Iranian Academic Research: A Systematic Review Using the Kitchenham & Charters Model." *International Journal of Information Science and Management* 21(3).
- Kierfeld, J., and V. M. Vinokur. 2006. "Slow Crack Propagation in Heterogeneous Materials." *Physical Review Letters* 96(17). doi:10.1103/PhysRevLett.96.175502.
- Kozlov, Vladimir V., Igor M. Dolganov, and Stepan S. Slobodin. 2024. "Selection of Numerical Method for Solving Ordinary Differential Equation Systems for a High-Speed Model of Hydrocarbons Steam Cracking." *Bulletin of the Tomsk Polytechnic University, Geo Assets Engineering* 335(1). doi:10.18799/24131830/2024/1/4364.
- Kushwaha, N., and V. N. Patel. 2023. "Nonlinear Dynamic Analysis of Two-Disk Rotor System Containing an Unbalance Influenced Transverse Crack." *Nonlinear Dynamics* 111(2):1109–37. doi:10.1007/s11071-022-07893-7.
- Li, K., and F. Wang. 2024. "A Combined Displacement Discontinuity-Interaction Integral Method for Computing Stress Intensity Factors and T-Stress." *International Journal of Solids and Structures* 301. doi:10.1016/j.ijsolstr.2024.112964.
- Li, W., S. Lin, Z. Wang, H. Guo, and X. Yu. 2024. "An Explicit Improved Meshless Numerical Manifold Method for Dynamic Crack Propagation." *Theoretical and Applied Fracture Mechanics* 130. doi:10.1016/j.tafmec.2024.104293.
- Liu, Quansheng, Lei Sun, Xuhai Tang, and Lei Chen. 2018. "Simulate Intersecting 3D Hydraulic Cracks Using a Hybrid 'FE-Meshfree' Method." *Engineering Analysis with Boundary Elements* 91. doi:10.1016/j.enganabound.2018.03.005.
- Liu, Xufei, Hai Guo, Lewei Zeng, Xiaopu Lyu, Yu Wang, Yangzong Zeren, Jin Yang, Luyao Zhang, Shizhen Zhao, Jun Li, and Gan Zhang. 2021. "Photochemical Ozone Pollution in Five Chinese Megacities in Summer 2018." *Science of the Total Environment* 801. doi:10.1016/j.scitotenv.2021.149603.
- Lv, Jia He, Yu Yong Jiao, Timon Rabczuk, Xiao Ying Zhuang, Xia Ting Feng, and Fei Tan. 2020. "A General Algorithm for Numerical Integration of Three-Dimensional Crack Singularities in PU-Based Numerical Methods." *Computer Methods in Applied Mechanics and Engineering* 363. doi:10.1016/j.cma.2020.112908.
- Mangaroo-Pillay, Mia, and Rojanette Coetzee. 2022. "Lean Frameworks: A Systematic Literature Review (SLR) Investigating Methods and Design Elements." *Journal of Industrial Engineering and Management* 15(2). doi:10.3926/jiem.3677.
- Markov, A., and S. Kanaun. 2018. "An Efficient Numerical Method for Quasi-Static Crack Propagation in Heterogeneous Media." *International Journal of Fracture* 212(1). doi:10.1007/s10704-018-0284-9.

- Mate-Kole, Emmanuel Matey, Dmitri Margot, and Shaheen Azim Dewji. 2023. "Mathematical Solutions in Internal Dose Assessment: A Comparison of Python-Based Differential Equation Solvers in Biokinetic Modeling." *Journal of Radiological Protection* 43(4). doi:10.1088/1361-6498/ad0409.
- Mohammed, Omar D., and Matti Rantatalo. 2016. "Dynamic Response and Time-Frequency Analysis for Gear Tooth Crack Detection." *Mechanical Systems and Signal Processing* 66–67. doi:10.1016/j.ymssp.2015.05.015.
- Mutra, Rajasekhara Reddy, D. Mallikarjuna Reddy, and K. Babu Rao. 2025. "Crack Fault Diagnosis in Rotor Bearing System by Transient and Study State Time Domain Analysis." *Measurement: Journal of the International Measurement Confederation* 241. doi:10.1016/j.measurement.2024.115667.
- Orie, O. U., and U. K. Ogbonna. 2024. "NUMERICAL METHOD FOR PREDICTING FIRST CRACK LOAD ON HOLLOW-CORED BEAMS SUBJECTED TO POINT LOAD AT MIDSPAN." *Nigerian Journal of Technology* 43(1):44–50. doi:10.4314/njt.v43i1.6.
- Païdoussis, Michael P. 2022. "Pipes Conveying Fluid: A Fertile Dynamics Problem." *Journal of Fluids and Structures* 114. doi:10.1016/j.jfluidstructs.2022.103664.
- Paul, Sumit. 2021. "Finite Element Analysis in Fused Deposition Modeling Research: A Literature Review." *Measurement: Journal of the International Measurement Confederation* 178. doi:10.1016/j.measurement.2021.109320.
- Qu, W., Y. Gu, and C. M. Fan. 2024. "A Stable Numerical Framework for Long-Time Dynamic Crack Analysis." *International Journal of Solids and Structures* 293. doi:10.1016/j.ijsolstr.2024.112768.
- Ri, Chol Uk, O. RyongSik, Qiang Zhao, Chung Hyok Chae, Yong Il Sin, and Yongkon Sin. 2022. "Dynamic Analysis of the Single Rotor-Bearing System Considering the Comprehensive Stiffness and Damping." *Journal of Mechanics* 38. doi:10.1093/jom/ufac022.
- Sarıtaş, Mustafa Tuncay, and Kıvanç Topraklıkoğlu. 2022. "Systematic Literature Review on the Use of Metaverse in Education." *International Journal of Technology in Education* 5(4). doi:10.46328/ijte.319.
- Scamardo, Manuela, Alberto Franchi, and Pietro Crespi. 2022. "A Non-Standard Numerical Method for Finite Element Modelling of Tensile Cracks in Quasi-Brittle Material." *Computers and Structures* 258. doi:10.1016/j.compstruc.2021.106664.
- Shahzamanian, M. M., Meng lin, Muntaseer Kainat, Nader Yoosef-Ghods, and Samer Adeeb. 2021. "Systematic Literature Review of the Application of Extended Finite Element Method in Failure Prediction of Pipelines." *Journal of Pipeline Science and Engineering* 1(2). doi:10.1016/j.jpse.2021.02.003.
- Shahzamanian, M. M., Amir Partovi, and P. D. Wu. 2020. "Finite Element Analysis of Elastic–Plastic and Fracture Behavior in Functionally Graded Materials (FGMs)." *SN Applied Sciences* 2(12). doi:10.1007/s42452-020-03901-w.
- Shlyannikov, V., D. Fedotova, and R. Khamidullin. 2023. "Mixed-Mode Crack Growth Analysis Using a Cyclic Plasticity Model." *Theoretical and Applied Fracture Mechanics* 128. doi:10.1016/j.tafmec.2023.104136.
- Shoheib, Mohammad M., Shahram Shahrooi, Mohammad Shishehsaz, and Mahdi Hamzehei. 2022. "Bézier Base Extended Isogeometric Numerical Method for Thermo Elastic-Plastic Analysis of Crack Propagation in Cracked Plate under Welding Residual Stress and Thermal Load." *Mathematical Modelling and Analysis* 27(4). doi:10.3846/mma.2022.15503.

- Su, C., K. Cai, and Z. Xu. 2023. "A SFBEM–FEM Coupling Method for Solving Crack Problems Based on Erdogan Fundamental Solutions." *Journal of Engineering Mathematics* 138(1). doi:10.1007/s10665-022-10247-2.
- Tikhomirov, V. M. 2020. "Numerical Method for Determining a Stress Intensity Factor in the Case of Solids of Homogeneous and Heterogeneous Materials with a Crack." *Journal of Applied Mechanics and Technical Physics* 61(1). doi:10.1134/S0021894420010149.
- Wang, Haijie, Xintian Liu, Xiaolan Wang, and Yansong Wang. 2019. "Numerical Method for Estimating Fatigue Crack Initiation Size Using Elastic–Plastic Fracture Mechanics Method." *Applied Mathematical Modelling* 73. doi:10.1016/j.apm.2019.04.010.
- Wang, Yi, Jinghua Ji, Wenxiang Zhao, Donghui Cao, and Zhujin Ren. 2022. "Meshless Generalized Finite Difference Method to Analyze Electromagnetic Performance of SPM Machines With Eccentric Rotor Shape." *IEEE Transactions on Industrial Electronics* 69(12). doi:10.1109/TIE.2021.3131872.
- Wang, Z., W. Tang, X. Li, Y. Yang, and Y. Bi. 2024. "Micro-Crack in Solids Evaluation Based on Zero-Frequency Component of the Critically Refracted Longitudinal Wave." *Measurement Science and Technology* 35(9). doi:10.1088/1361-6501/ad56ba.
- Wang, Zhengjun, Xinyang Liu, Chen Dong, Jie Chen, and Lianxiang Liu. 2023. "Thermal Fatigue Crack Propagation Process and Mechanism of Multicomponent Al-7Si-0.3Mg Alloy." *Crystals* 13(7). doi:10.3390/cryst13071068.
- Zedan Khalel, Hamad Hasan, Muhammad Khan, and Andrew Starr. 2023. "Dynamic Response-Based Crack Resistance Analysis of Fibre Reinforced Concrete Specimens under Different Temperatures and Crack Depths." *Journal of Building Engineering* 66. doi:10.1016/j.jobbe.2023.105865.
- Zhang, Zhennan, and Xiurun Ge. 2008. "Virtual Multi-Dimensional Internal Bonds Model and Its Application in Simulation of Rock Mass." *Science in China, Series E: Technological Sciences* 51(2). doi:10.1007/s11431-008-0014-z.
- Zhao, Junliang, and Dongxiao Zhang. 2020. "Dynamic Microscale Crack Propagation in Shale." *Engineering Fracture Mechanics* 228. doi:10.1016/j.engfracmech.2020.106906.
- Zhao, Youxuan, Yongmei Xu, Zimu Chen, Peng Cao, and Ning Hu. 2018. "Detection and Characterization of Randomly Distributed Micro-Cracks in Elastic Solids by One-Way Collinear Mixing Method." *Journal of Nondestructive Evaluation* 37(3). doi:10.1007/s10921-018-0505-1.