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DIFFERENTIAL EQUATION LEARNING ENGAGEMENT EXPERIENCE AMONG THE CIVIL ENGINEERING STUDENTS USING COMPUTER ALGEBRA SYSTEM

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Abstract:

This article discusses a novel instructional method for a civil engineering course focused on solving a spring-mass system's differential equation (DE). The Computer Algebra System (CAS), MAXIMA, is used by our approach to prioritize practical learning, especially in solving ordinary differential equations. By ensuring minimal programming effort, we designed MAXIMA applications to enhance the learning experience, enabling intuitive investigation of the spring-mass system by students. Student belief in solving DE is significantly enhanced by this method, as indicated by our study, in comparison to traditional approaches. Incorporating CAS, these findings offer crucial insights into creating innovative educational strategies for engineering students.

Keywords:

Spring-Mass, Computer Algebra System, MAXIMA, Differential Equation

Introduction

The ability to solve differential equations (DE), a specialized mathematics discipline, is crucial for future civil engineers to describe how real-world civil engineering problems respond to the natural forces. Research indicates that many future engineers struggle with mathematical



disciplines and find it challenging to connect mathematics with their engineering applications from their first year of study (Panaoura et al.,2024; Charalambides et al., 2023; Le & Alan, 2019). Success in solving DE problems requires more than just knowledge of algorithms, basic mathematical principles, and procedures. Emotional states, such as frustration, stress, anxiety (Cabrera et al., 2024), and mood, as well as beliefs about tackling mathematical tasks, significantly influence the process. A study by Gopal et al. (2020) among upper secondary school students in Malaysia supported Cabrera's findings, showing that these students had low confidence in their learning abilities and were concerned about achieving good grades in mathematics and may affect their mathematical solving capability. It is assumed that a similar trend applies to Malaysian undergraduate students taking more advanced mathematics courses, as some may have harboured these concerns since their time in secondary school.

Among these factors, beliefs are the most crucial components because they serve as the reasons for learning mathematics and generally create motivations that define the learning context for mathematics (Aisha et al., 2017). The relationship between belief about mathematics and mathematical performances has been studied by McLoed (1992). He shares the viewpoint that beliefs about mathematics can either boost or diminish an individual's mathematical and problem-solving abilities. To cultivate and drive appropriate belief state, Scott et al. (2022) had utilized the Computer Algebra System (CAS) technology to investigate belief stability among senior secondary mathematics students in Australia. Additionally, CAS can aid in mathematical visualization via graphical features, automate tasks, and provide multiple representations of concepts (Shé et al., 2023), especially in a complex topic which promotes another learning engagement experience among the civil engineering students. Student engagement has been widely researched and found to be linked positively with the student belief values on solving capabilities in their subject.

Therefore, this work is inspired by the studies of Scott et al. (2022), Aisha et al. (2017), and Moodi et al. (2021). Motivated by Scott's research, we hypothesized that the belief values of the sixteen students enrolled in the Numerical Analysis for Engineers course (CES 513) at the undergraduate level in Civil Engineering Studies at Universiti Teknologi MARA Cawangan Pulau Pinang will be key factors in solving differential equation (DE) problems, such as the spring-mass system, using CAS technology through MAXIMA software.

Literature Review

There are two important points that will be discussed in this Literature Review section.

Issues in Learning DE and The Implementation of CAS

The development of a conceptual and relational understanding of mathematical and engineering applications through specific pedagogical tools has garnered significant interest from engineering researchers and educators worldwide. One such tool is the integration of digital technologies into the engineering curriculum to enhance the comprehension of differential equations (DE) among future civil engineers (Haleem et al., 2022). Educators can employ innovative teaching and learning styles to meet the diverse needs of students, particularly those with unstable beliefs about their ability to solve complex mathematical problems (Mason, 2003; Abedalaziz & Zamri, 2012). The use of digital technologies, such as Computer Algebra System (CAS) software packages, for solving complex DE problems has shown potential in altering the attitudes and motivations of demotivated students. These tools



Volume 7 Issue 24 (March 2025) PP. 577-588 DOI: 10.35631/IJMOE.724040 facilitate predictions and offer a deeper understanding of modelling real-world civil engineering phenomena, such as the spring-mass system.

CAS software packages, whether acquired through a purchased license or as open-source software, have been widely used for manipulating numerical and symbolic mathematical expressions, as well as for graphing and programming. Prominent examples of CAS software available in the market include MAPLE, Mathematica, and MATLAB. These tools are employed in university courses, research, and various other applications. However, MAXIMA, developed in the late 1960s at the Massachusetts Institute of Technology, offers the distinct advantage of being free to install, as it is released under the GNU General Public License (Zakova, 2014). This cost-free accessibility gives MAXIMA an edge over the three well-known CAS software packages

Spring-mass System

Civil engineers are deeply concerned with the vibrational behaviours of infrastructure elements such as bridges, buildings, dams, tunnels, and other structures during their operational lifetimes. Mathematical models are typically developed to measure the degrees of freedom in motion and position of these structures over time t. For instance, the motion of a reinforced concrete building structure can be modelled using springs, dashpots, and masses, as depicted in Figure 1.

Sectional view of building

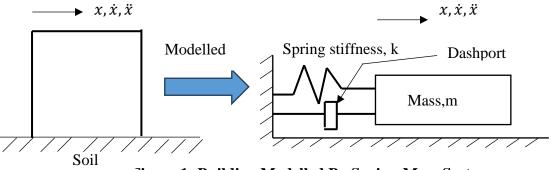


Figure 1: Building Modelled By Spring-Mass System

The general spring-mass system equation is expressed as Equation (1) (Cruzz et al., 2012):

$$m\ddot{x} + c\dot{x} + x = f(t) \tag{1}$$

where *m*, *c* and *k* is the lumped mass of the building system, *c* is the damping coefficient of the building system and k is the spring constant of the building system. f(t) is the force acting on the building structures, and called as free vibration problem when f(t) = 0.

Solving Equation (1) to determine the position, x, velocity, \dot{x} and acceleration, \ddot{x} of the mass requires fundamental knowledge of integral and differential calculus, as well as linear algebra. Civil engineering students who lack proficiency in these prerequisite subjects may find solving this equation challenging. To support these students, specific learning styles and motivational strategies are necessary to enhance their long-term learning outcomes.



Methodology

A sequential approach consisting of three distinct stages, illustrated in Figure 2, was employed in this work for sixteen students of two different groups of academic ability. In the initial stage, data on the participants' proficiency in solving DE based on their acquired skills from the previous semester was gathered. DE as given Table 1 were utilized for this purpose in solving two different problems of spring-mass free vibration of undamped and damped system, respectively. Problem number 1 involves the mathematical modelling of an undamped spring-mass system, where a mass of 2 kg is attached to a spring with a stiffness of 128 kN/m. In contrast, Problem 2 models a damped vibration system, incorporating a damper with a damping coefficient of 128 Nm/s, while the mass and spring stiffness remain the same. In both problems, the mass is initially displaced 0.2 m from its equilibrium position. However, in Problem 1, the initial velocity is 0 m/s, whereas in Problem 2, the initial velocity is 0.6 m/s. The sample manual solutions developed by the students in this stage will serve as a basis for comparison in subsequent stages and shown in Figure 3a and Figure 3b.

Problem No.	DE needs to be solved	
1	$2\ddot{x} + 128x = 0, \qquad x(0) = 0.2, \dot{x}(0) = 0$	
2	$2\ddot{x} + 40\dot{x} + 128x = 0$, $\dot{x}(0) = 0.2$, $\dot{x}(0) = 0.6$	

where \ddot{x}, \dot{x} and x represent the $\frac{d^2x}{dt^2}$ or the acceleration, $\frac{dx}{dt}$ or the velocity and x(t), the position of the vibrating mass when self-excited at time t. $\dot{x}(0)$ and x(0) denotes the velocity and position of vibrating mass when self-excited at zero second.

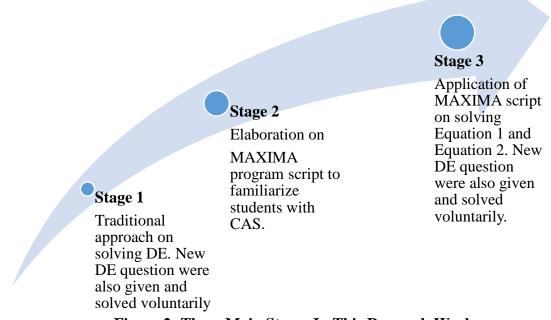


Figure 2: Three Main Stages In This Research Work



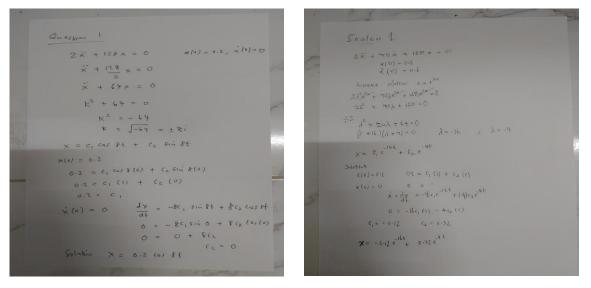


Figure 3a (left) and 3b (right): Sample Manual Solution for Problem 1 and 2

In the first stage of the study, sixteen students enrolled in the CES513 course were selected as participants. These students were divided into two groups based on their performance in their previous Differential Equations course. Group A consisted of students with weak performance (grades below or equal to C), while Group B comprised students with high performance (grades B+ or higher). Prior to any assessments, the students received a general tutorial on the free-vibration problem in undamped and damped systems modelled as spring-mass systems. The variables and physical parameters related to Differential Equations (DE) listed in Table 1 were thoroughly explained to prevent any technical misunderstandings or confusion about the mathematical concepts.

After this preliminary explanation, each student was tasked with providing a step-by-step handwritten solution to the DE in Table 1, along with handwritten graphical plots, within one hour. Participants were then given a fifteen-minute break before proceeding to the second stage of the study. In this stage, all students were introduced to the basic features of the Computer Algebra System (CAS) software MAXIMA by examining pre-programmed scripts designed to solve the equations given in stage 1. Additionally, new DE problems, as shown in Table 2, were presented to the participants, who were encouraged to solve them voluntarily.

In the third stage, once all participants had familiarized themselves with the MAXIMA scripts and understood the solution process using the CAS approach, they were given the same amount of time as in stage 1 to solve the designated problems. Students who completed all assigned tasks within this time frame were invited to voluntarily work on additional DE problems related to the spring-mass system, as provided in Table 2.



No of new question	Form of question	
1	A mass weighing 0.91 kg stretches a spring	
	0.15 m. At $t = 0$ the mass is released from a	
	point 0.2 m below the equilibrium position	
	with an upward velocity of 0.91 m/s.	
	Determine and draw the equation of motion.	
2	A mass weighing 3.6 kg, attached to the end	
	of a spring, stretches it 2.44 m. Initially, the	
	mass is released from a point 0.15 m below	
	the equilibrium position with a downward	
	velocity of 0.46 m/s. Find and draw the	
	equation of motion.	
3	A1-kilogram mass is attached to a spring	
	whose constant is 16 N/m, and the entire	
	system is then submerged in a liquid that	
	imparts a damping force numerically equal to	
	10 times the instantaneous velocity.	
	Determine the equations of motion if the	
	mass is initially released from rest from a	
	point 1 meter below the equilibrium position.	

Table 2: Voluntary Problem-Solving After Completing DE in Table 1

Notably, all stages were conducted within the same day, approximately within a three-hour period. All activities were guided using a predefined MAXIMA script tailored to align with the research objectives.

Findings and Discussion

Traditional/Manual DE Solution Answer Script Analysis

In the first stage of the study, participants were prohibited from using any external assistance from textbooks, websites, or other sources to provide manual solutions to the Differential Equation (DE) presented in Table 1. They were guided solely by their prior knowledge and memory from the given tutorial session. Despite this guidance, only 12.5 percent of the participants in Group A were able to provide complete and correct procedural steps to solve the DE in Table 1. In contrast, a remarkable 87.5 percent of participants in Group B demonstrated proficient solving capabilities. The high-performing students in Group B appeared to enjoy the challenge as the questions became progressively more difficult, due to the inclusion of the damped condition to the problem. When presented with application questions of ODEs in engineering problems, one of the high-performing students successfully obtained the correct solution. The reluctantly to solve new voluntarily DE problem were apparently shown by all of participants in Group A, whereas 75 percent of participant from Group B tried to solve at least two DE from Table 2.

It is hypothesized that Group A's lower DE solving capability compared to Group B may be primarily influenced by a "low motivation" feeling, stemming from their previous low marks in their past Differential Equations subject. These findings were anticipated by the authors and are consistent with the works of McLeod (1992), Gopal et al. (2020), and Cabrera et al. (2024).



CAS Experience Solution Learning

In the second and third stages, participants conducted a Computer Algebra System (CAS) session using the open-source MAXIMA software. Leveraging the step-by-step knowledge acquired from examining two pre-programmed MAXIMA scripts, the authors instructed students to manually input the provided commands from a printed sheet to produce similar results. The commands for solving differential equations (DE) presented in Table 1 were demonstrated to the participants during the second stage session, as detailed in Table 3. The outputs for undamped and damped DE problems are illustrated in Tables 4 and 5, respectively.

(Undamped Problem)			
Description	MAXIMA commands		
Delete all variables from memory.	-> kill(all)\$		
Define the variables involved in the DE formulation.	-> depends (x,t)\$;		
Establish the DE symbolically.	-> eq1:2. diff(x, t,2) + $128.x = 0$;		
Symbolically solved the DE & to provide general solution	-> ode2(%, x,t)		
Find the constant by substitute the initial condition to the solution & provide the particular solution of DE.	-> ic2(%,t=0, x=0.2, ;diff(x,t) =0);		
Plot the solution of DE and configure plotting scaling and the label for horizontal and vertical.	<pre>-> wxdraw2d(explicit (1/5*(cos(8*t)), t,0,1.5), xlabel="t", ylabel ="Amplitude",grid = [1,1]);\$</pre>		

Table 3: Description of each MAXIMA Script Command To Solve DE In Table 1 (Undamped Problem)

Table 4: Output Of The Execution Of Each MAXIMA Script Command (Undamped)
Problem)

MAXIMA command	Output
-> kill(all)\$	Not appear as the output is suppressed by \$ symbol
\rightarrow depends (x,t)\$;	Not appear as the output is suppressed by \$ symbol
-> eq1:2. diff(x, t,2) + 128.x = 0;	Not appear as the output is suppressed by \$ symbol
	$x = \%k1 \cdot \sin(8 \cdot t) + \%k2 \cdot \cos(8 \cdot t)$
-> ode2(%, x,t)	
-> ic2(%,t=0, x=0.2, ;diff(x,t) =0);	$x = \cos(8 \cdot t)/5$



DOI: 10.35631/IJMOE.724040 0.2 -> wxdraw2d(explicit $(1/5^*(\cos(8^*t)))$, 0.15 xlabel="t", t,0,1.5), ylabel 0.1 ="Amplitude",grid = [1,1]);\$ 0.05 0 mplit -0.05 -0.1 -0.15 -0.2 0.2 0.4 0.6 0.8 1.2 1.4

Table 5: Output Of The Execution Of Each MAXIMA Script Command (Damped Problem)

MAXIMA command	Output		
-> kill(all)\$	Not appear as the output is suppressed by \$ symbol		
-> depends (x,t)\$;	Not appear as the output is suppressed by \$ symbol		
-> eq2:2*diff(x,t,2) +40*diff(x,t) + 128*x =0\$	Not appear as the output is suppressed by \$ symbol		
-> ode2(%, x,t)	$x = \% k1 \cdot \% e^{(-4 \cdot t)} + \% k2 \cdot \% e^{(-16 \cdot t)}$		
<pre>-> ic2(%,t=0,x=0.2,'diff(x,t)=0.6); -> wxdraw2d(explicit (1/60*(19*exp(-4*t)-7*exp(-16*t)), t,0,1.5), xlabel="t", ylabel</pre>	$x = (19 \cdot \% e^{(-4 \cdot t)})/60 - (7 \cdot \% e^{(-16 \cdot t))}/60$		
="Amplitude",grid = [1,1]);\$	0.05		
	0 0.2 0.4 0.6 0.8 1 1.2 1.4 t		

During this stage, Group A and Group B exhibited different responses compared to the first stage. Initially, all participants enjoyed solving the DEs listed in Table 1 using the CAS approach, irrespective of their prior DE-solving abilities. During this activity, students were asked to compare their handwritten solutions with the MAXIMA output. All participants, particularly those in Group A, demonstrated a positive attitude by voluntarily solving at least one of the additional questions provided in Table 2. Students also learned about the



characteristics of DE solutions through the graphical plots generated by the MAXIMA script, gaining an understanding of the differences between undamped and damped spring-mass systems.

Additionally, the distinctions between general and particular solutions of DEs were highlighted by comparing the solutions presented in rows 3 and 4 of Table 3. This process not only enhanced the understanding of low-performing students but also provided them with a new dimension for learning DE problems.

Comparison Of Engagement Learning Experience Between Traditional And CAS Approach The comparison learning engagement experience for both groups between the traditional and the CAS approach is summarized in Table 6 and Figure 4.

Table 6: Comparison	Of Learning	g Engagement H	Experience For	Both Groups
		,		

Approach	Group A	Group B
Traditional DE Solution	1 student (12.5 %)	7 students (87.5 %)
CAS approach	8 students (100 %)	8 students (100 %)

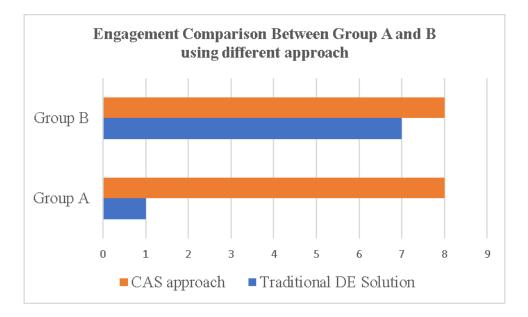


Figure 4: Visual Comparison Of Learning Engagement Experience For Both Groups

Table 6 suggest that 1 student (12.5%) from Group A successfully solved the DE using the traditional method. This reflects a significant struggle among participants in this group when relying solely on their prior knowledge and manual computation. The low success rate could be due to several factors, such as inadequate foundational understanding, lack of confidence, or insufficient exposure to solving DEs without external aids. The minimal success suggests that this group may have a weaker grasp of the mathematical concepts or problem-solving techniques required for DEs. In stark contrast, 7 out of 8 students (87.5%) in Group B demonstrated success in the traditional approach. This high percentage indicates a much stronger command of DEs, possibly resulting from a more solid foundation in mathematical principles and a higher level of engagement with the subject matter. The students in Group B



seem better equipped to handle complex DE problems manually, suggesting that they have mastered the theoretical aspects and procedural steps needed to solve DEs without the aid of software.

When transitioning to the CAS approach, all 8 students (100%) in Group A successfully solved the DEs. This dramatic improvement suggests that Group A greatly benefited from the use of the CAS software, which provided them with computational assistance and visual tools that may have compensated for their initial difficulties. The interactive nature of the CAS, along with its graphical output, likely made the mathematical concepts more accessible and less intimidating, fostering increased engagement and understanding. It is also possible that the CAS approach minimized the cognitive load of manual calculations, allowing students to focus more on the conceptual understanding of DEs. Similarly, all 8 students (100%) in Group B succeeded in the CAS approach, which is consistent with their performance in the traditional method. This reinforces the conclusion that Group B had a strong understanding of DEs to begin with. However, the CAS may have still added value by offering additional insights through graphical visualization and automatic solution verification, further enhancing their learning experience. While Group B did not necessarily "need" the CAS for success, it likely enriched their problem-solving experience by allowing them to explore DE solutions more efficiently and comprehensively.

Conclusions

At the outset of the study, participants were required to solve DE without relying on external resources such as textbooks or online materials. In this phase, Group A exhibited a low success rate, with only 15% of participants completing the DEs with accurate procedural steps. Conversely, Group B displayed a significantly higher success rate, achieving 87.5%. The participants in Group B, particularly the high performers, embraced the challenge, especially when tasked with more complex problems involving damped conditions. One top student in Group B effectively solved a DE related to an engineering application of Ordinary Differential Equations (ODEs). Meanwhile, the majority of Group A participants refrained from voluntarily attempting new DE problems, whereas 75% of Group B participants attempted at least two additional problems beyond the initial requirement.

The disparity in performance between the two groups may be attributed to motivational differences. Group A's lack of enthusiasm could stem from previous poor performance in DE-related subjects, a finding that resonates with research by McLeod (1992), Gopal et al. (2020), and Cabrera et al. (2024).

In subsequent stages, both groups participated in sessions using the open-source MAXIMA software, where they replicated results by manually inputting commands from a pre-printed sheet. Students, regardless of prior ability, initially enjoyed solving the DEs outlined in Table 1 through this approach, showing their engagement learning behaviour to this topic. The activity allowed participants to compare their handwritten solutions with those produced by MAXIMA, leading to voluntary engagement with additional problems, particularly among Group A participants. The software-generated plots visually demonstrated the nature of the DE solutions, particularly emphasizing the distinctions between damped and undamped spring-mass systems.



The use of MAXIMA also highlighted the contrast between general and particular solutions, as seen in the solutions presented in Table 3. This hands-on, visual approach appeared to deepen students' understanding of DEs, especially for those who previously struggled. Overall, the CAS experience proved to be a valuable pedagogical tool, fostering improved engagement experience and comprehension of DEs through interactive learning in any other topics than DE.

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